

# Namaste



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**Jonathan J. Crabtree, Elementary Mathematics Historian, Australia**  
**With grateful appreciation to Dr. Inder K. Rana.**



# **PART 1. Why India Must Change its Story**

## **Bad Maths History $\Rightarrow$ Sad Maths Misery**



# **PART 2. Negative & Positive Quantities on a**

## **Brahmaguptan Plane for India's Primary Classes**



# **PART 1. Why India Must Change its Story**

## **Bad Maths History $\Rightarrow$ Sad Maths Misery**



**Jonathan J. Crabtree, Elementary Mathematics Historian, Australia.**

**9th National conference: Technology & Innovations in Math Education**

**Biennial Conference of the Mathematics Department of the IIT Bombay.**

**Jointly organized by IISER, Pune & BATU, Lonere.**

**Pune India, 27<sup>th</sup> December 2019**



I couldn't make much sense of what  
I was being taught. So I felt stupid.



I couldn't make much sense of what  
I was being taught. So I felt stupid.



“Don't worry  
about it, Jonathan.”



I couldn't make much sense of what  
I was being taught. So I felt stupid.

Mathematics is only  
confusing if you think  
about it.”



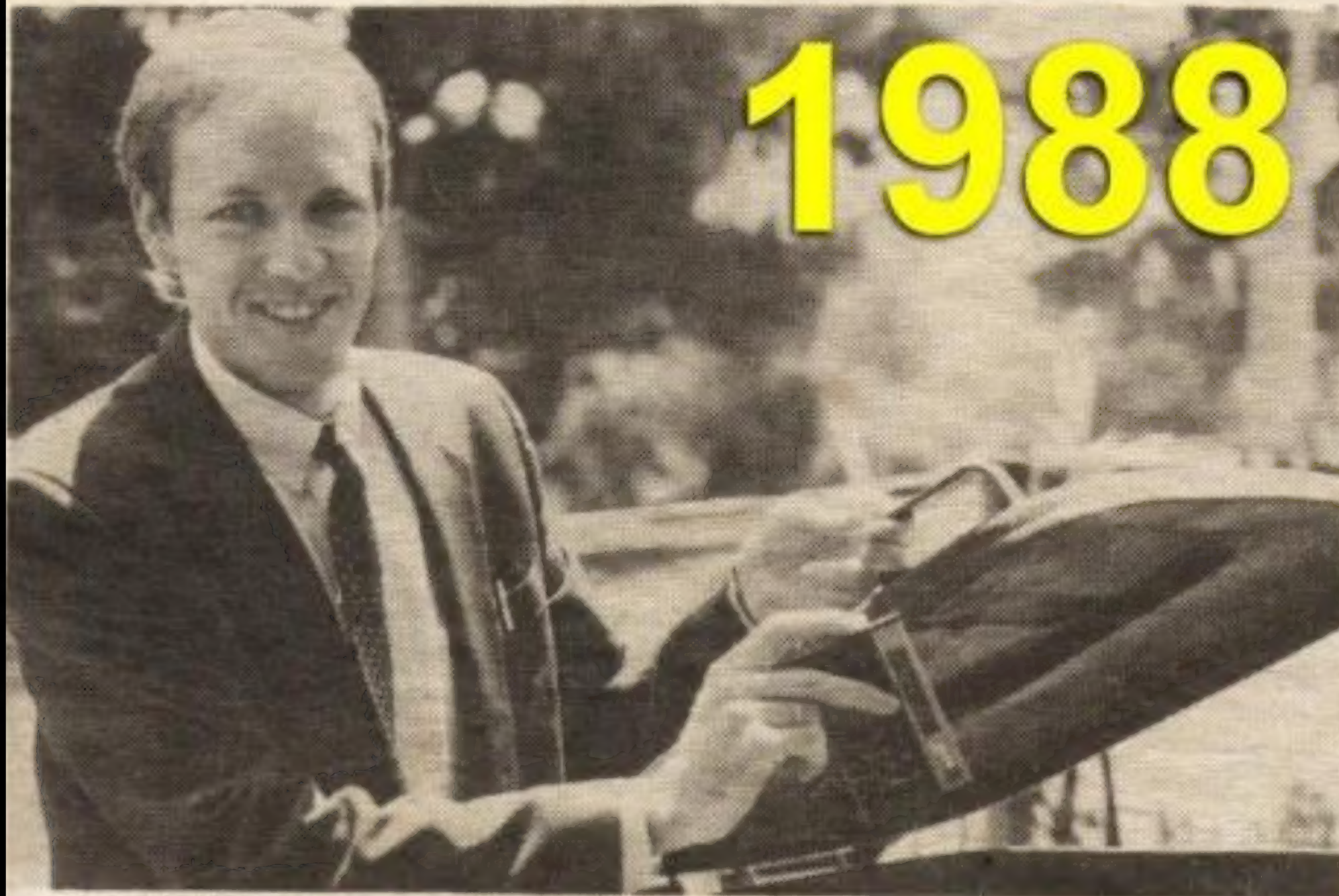
So, in 1983 I made a decision to change primary  
and middle school mathematics explanations.



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Maths? It's all in the mind, says Jonathan

1988



JONATHAN throws away his calculator and uses brain powers to solve even the hardest of equations.

IF you were asked what day it was on July 24, 1706, what would you say?

It's all in the mind, he says.

After a four second calculation he came up with the correct day.

Jonathan broke the world record for

"I hope to change the way the Western world teaches maths," Jonathon said.

Jonathon will be holding two classes at the Park Orchards Community Centre.

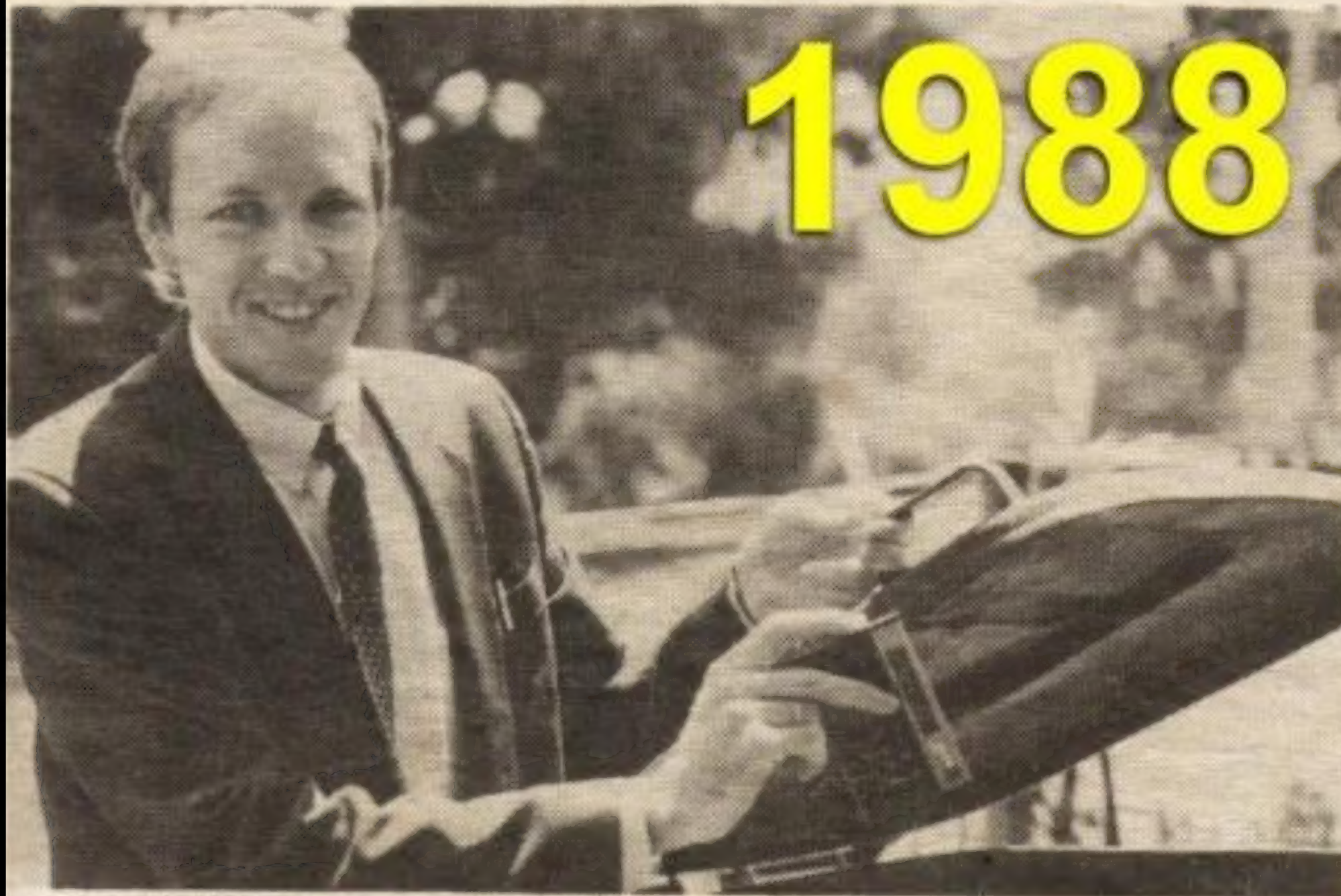
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At age seven in Class 2, I pulled on a loose thread.



The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

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The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

So, that classroom confusion in 1968  
led to a peer-reviewed paper.



# THE LOST LOGIC OF ELEMENTARY MATHEMATICS AND THE HABERDASHER WHO KIDNAPPED KAIZEN

**Jonathan Crabtree**    **Download the paper @** [www.bit.ly/LostLogicOfMath](http://www.bit.ly/LostLogicOfMath)

*www.jonathancrabtree.com | Mathematics Historian*

*Euclid's multiplication definition from Elements, (c. 300 BCE), continues to shape mathematics education today. Yet, upon translation into English in 1570 a 'bug' was created that slowly evolved into a 'virus'. Input two numbers into Euclid's step-by-step definition and it outputs an error. Our multiplication definition, thought to be Euclid's, is in fact that of London haberdasher, Henry Billingsley who in effect kidnapped kaizen, the process of continuous improvement. With our centuries-old multiplication definition revealed to be false, further curricular and pedagogical research will be required. In accordance with the Scientific Method, the Elements of western mathematics education must now be rebuilt upon firmer foundations.*



At age seven in Class 2, I pulled on a loose thread.

The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

So, that classroom confusion in 1968  
led to a peer-reviewed paper.

**The more I pulled on loose threads, the  
more mathematics education unraveled.**



# INDIA WE HAVE A PROBLEM



## Many Dislike or Fear Your Maths



# INDIA WE HAVE A PROBLEM



Many Dislike or Fear Your Maths  
**What happened?**





**You can't cross the sea  
merely by standing and  
staring at the water.**

Rabindranath Tagore



**I must travel to find out if there is demand for India's original and true foundations of mathematics.**

**You can't cross the sea merely by standing and staring at the water.**

Rabindranath Tagore



















# Arithmetic?

## Basic Operations (+, −, ×, ÷) on



# Arithmetic?

Basic Operations (+, −, ×, ÷) on

... −3 −2 −1 0 +1 +2 +3 ...



**Obviously, India's ancient  
integer logic got to us today.**



Obviously, India's ancient  
integer logic got to us today.

**But how?**



***What FACTS do we know?***



# ***What FACTS do we know?***

India's definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN



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India's definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which featured Brahmagupta's ancient laws of sign for negatives and positives



# ***What FACTS do we know?***

India's definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which featured Brahmagupta's ancient laws of sign for negatives and positives

Based on what he learned from the Indians, al-Khwārizmī then wrote a book on algebra



# Al-Khwārizmī's **algebra** text c. 820 CE

- لكتاب المختصر في حساب الجبر والمقابلة
- Al-Kitāb al-mukhtasar fī hisāb **al-jabr** wa'l-muqābala
- The Compendious Book on Calculation by Completion and Balancing.



# *What FACTS do we know?*

Arabic writers understood **negative** terms. E.g.

In mathematical language, the verb [jabr] means...  
... to transpose **negative** quantities to the opposite side by changing their signs. The **negative** quantity thus removed...

Rosen 1831: p. 178



# *What FACTS do we know?*

Arabic writers understood **negative** terms. E.g.

The usual meaning of jabr in mathematical treatises is: adding equal terms to both sides of an equation in order to eliminate **negative** terms.

Van der Waerden 1985: p. 4



# *What FACTS do we know?*

Arabic writers understood **negative** terms. E.g.

Al-jabr means “restoration” or “completion”, that is, removing **negative** terms, by transposing them to the other side of the equation to make them positive

Devlin 2012: p. 53



# ***What FACTS do we know?***

From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them



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Leonardo Pisano then documented India's mathematical foundations involving Brahmagupta's definition of zero as a number



# ***What FACTS do we know?***

From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them

Leonardo Pisano then documented India's mathematical foundations involving Brahmagupta's definition of zero as a number

**Thus, Europe came to understand Indian arithmetic**



# ***The Transmission of India's Integer Arithmetic***

**India**

**7<sup>th</sup> century**





# *The Transmission of India's Integer Arithmetic*

**India**

**7<sup>th</sup> century**



**Arabic World**

**9<sup>th</sup> century**





# ***The Transmission of India's Integer Arithmetic***

**India**

**7<sup>th</sup> century**



**Arabic World**

**9<sup>th</sup> century**



**Europe**

**13<sup>th</sup> century**





# ***What FACTS do we teach?***

Zero is defined as  $n - n$



# ***What FACTS do we teach?***

Zero is defined as  $n - n$

Negative numbers are less than zero



# ***What FACTS do we teach?***

Zero is defined as  $n - n$

Negative numbers are less than zero

Negative seven is less than negative four  $-7 < -4$



# ***What FACTS do we know?***

Zero is defined as  $n - n$

Negative numbers are less than zero

Negative seven is less than negative four  $-7 < -4$

Every basic arithmetical operation (+, −, ×, ÷) on the Integers is understood and has been for centuries



# ***What FACTS do we teach?***

Euclid in his book *Elements* defined multiplication as repeated addition



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Euclid in his book *Elements* defined multiplication as repeated addition

***ab*** is thus defined as ***a*** added to itself ***b*** times



# ***What FACTS do we teach?***

Euclid in his book *Elements* defined multiplication as repeated addition

$ab$  is thus defined as  $a$  added to itself  $b$  times

$a^b$  is thus defined as  $a$  into itself  $b$  times



**What we know about  
arithmetic isn't true!**



**What we know about  
arithmetic isn't true!**

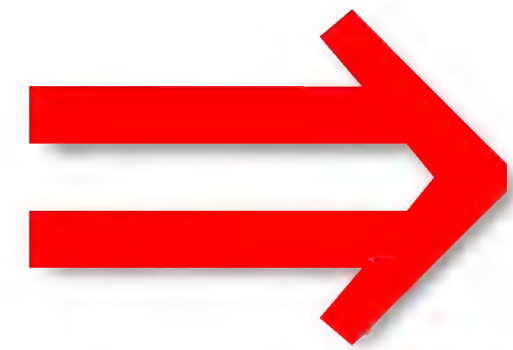
**Every previous  
'FACT' is FALSE!**



# Bad Maths History



# Bad Maths History



# Sad Maths Misery



# Indian students rank 2nd last in global test

TNN | Jan 15, 2012, 02.24 AM IST



*School students celebrate after checking their CBSE results. A global survey has found that the average 15-yea... [Read More](#)*

MUMBAI: Across the world, India is seen as an education powerhouse — based largely on the reputation of a few islands of academic excellence such as the IITs. But scratch the glossy surface of our education system and the picture turns seriously bleak.

Fifteen-year-old Indians who were put, for the first time, on a global stage stood second to last, only beating Kyrgyzstan when tested on their reading, math and science abilities.

India ranked second last among the 73 countries that participated in the Programme for International Student Assessment (PISA), conducted annually to evaluate education systems worldwide by the OECD (Organisation for Economic Co-operation and Development) Secretariat. The survey is based on two-hour tests that half a million students are put

through.

Tamil Nadu and Himachal, showpieces of India's education and development, fared miserably at the Programme for International Student Asssment, conducted by the Organisation for Economic Co-operation and Development Secretariat.

In math, considered India's strong point, they finished second and third to last, beating only Kyrgyzstan



# Bad Maths History

# Sad Maths Misery

*Everyone is entitled to their own opinion, yet not to their own facts.*



# Extraordinary Claims

- India's definition of **ZERO** never made it to either the ancient Arabic world or Europe.



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- India's definition of **ZERO** never made it to either the ancient Arabic world or Europe.
- In the Arabic world, India's **ZERO** only came to exist as a placeholder, not as the power tool to solve simple problems like  $+3$  minus  $+4$ , or  $-2$  minus  $-4$ , or  $-4$  minus  $+2$



# Extraordinary Claims

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***Extraordinary Evidence...***



# Brahmagupta

# 598 - 668 CE





**Brahmagupta**

**598 - 668 CE**



**Astronomer and Mathematician**



**Brahmagupta**

**598 - 668 CE**



**Brāhmasphuṭasiddhānta 628 CE**



For this talk,  
Brahmagupta's  
Laws of Positives  
Negatives and  
Zero have been  
freshly analysed.

Images courtesy of  
the British Library.

REPRINT FROM THE PANDIT.

ब्राह्मस्फुटसिद्धान्तो

ध्यानग्रहोपदेशाध्यायश्च ।

गणकचक्रचूडामणिश्रीब्रह्मगुप्तविरचितः ।

महामहोपाध्यायसुधाकरद्विवेदिकृतनूतन-  
तिलकसमेतः ।

BRĀHMASPHUṬASIDDHĀNTA

AND

DHYĀNAGRAHOPADEŚĀDHYĀYA,

BY BRAHMAGUPTA,

EDITED WITH HIS OWN COMMENTARY

BY

MAHĀMAHOPĀDHYĀYA SUDHĀKARA DVIVEDIN,

*Professor, Queen's College, Benares.*



BENARES:

PRINTED AT THE MEDICAL HALL PRESS.

1902.



# Brahmagupta's 5 Addition Laws AL (*saṅkalana*)



# Brahmagupta's 5 Addition Laws

**AL1** positive plus positive is positive

**AL2** negative plus negative is negative

**AL3** positive plus negative is the difference between the positive and negative

**AL4** when positive and negative are equal the sum is zero

**AL5** positive plus zero is positive  
negative plus zero is negative  
zero plus zero is zero



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# Brahmagupta's 5 Addition Laws

**AL1** positive plus positive is positive Understood in Arabic world

**AL2** negative plus negative is negative

**AL3** positive plus negative is the difference between the positive and negative

**AL4** when positive and negative are equal the sum is zero

**AL5** positive plus zero is positive  
negative plus zero is negative  
zero plus zero is zero





# ***Al-Khwārizmī (c. 780-850)***

I had seen that the Indians had set up **9 symbols** in their universal system of numbering...



# ***Al-Khwārizmī (c. 780-850)***

So they made **9 symbols**, which are these: 9 8 7 6 5 4 3 2 1.

And ... **every number is put together above one.**

**Al-Khwārizmī did not mention zero**

"Algorizmi said: since I had seen that the Indians had set up IX symbols..."  
Crossley, John N, and Henry, Alan S. (1990) *Thus Spake Al-Kḥwārizmī: A Translation of the Text of Cambridge University Library Ms. li. Vi. 5*. Historia Mathematica. P. 110-111

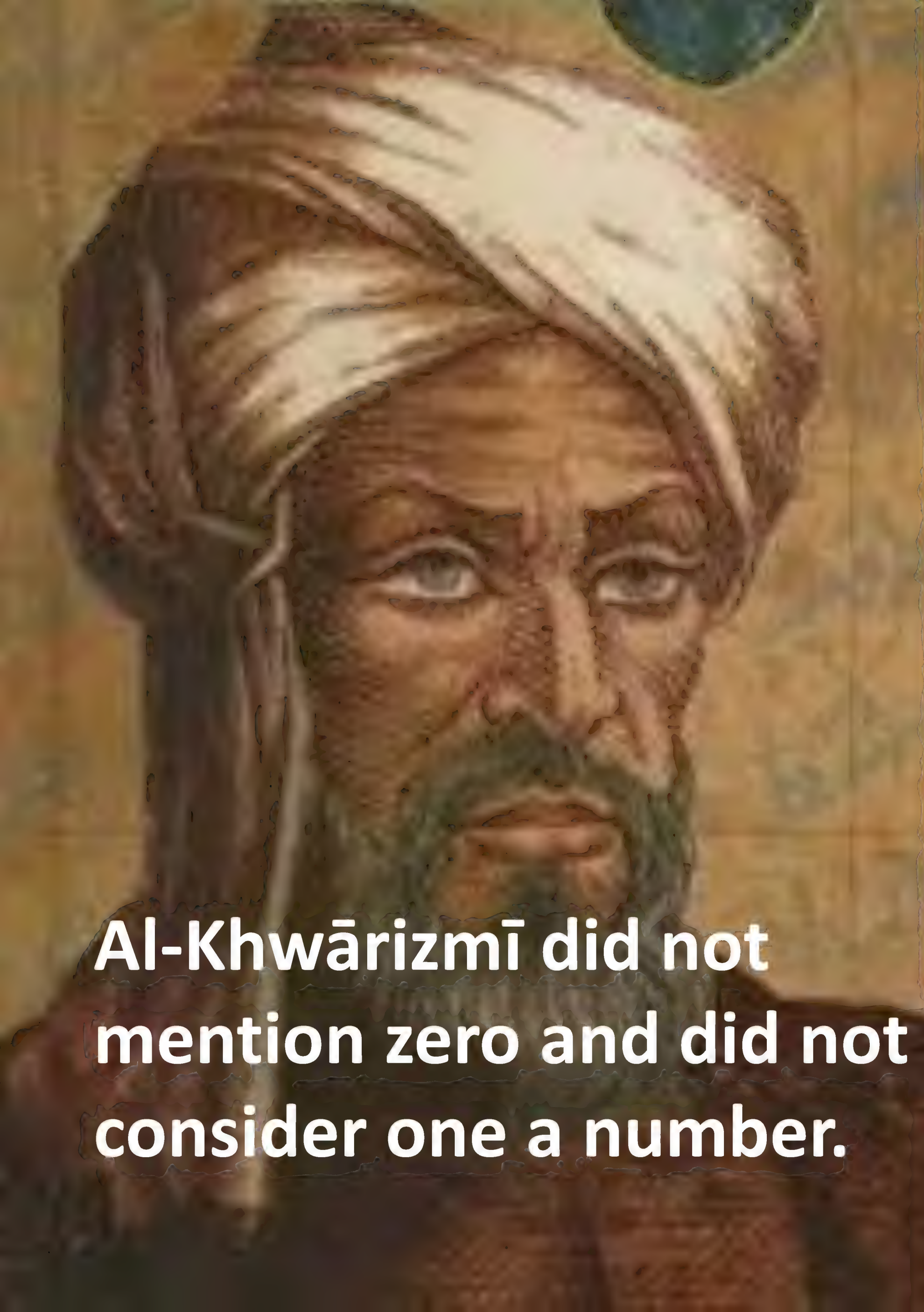


# ***Al-Khwārizmī (c. 780-850)***

... **one** is the root of all number and **is outside number**.

It is the root of number because every number is found by it.

But it [one] is outside number because it is found by itself, I mean, without any other number.

A portrait of Al-Khwārizmī, a Persian scholar, depicted with a beard and a turban, looking slightly to the right.

**Al-Khwārizmī did not mention zero and did not consider one a number.**



# ***Al-Uqlidisi (c. 920-980)***



**Al-Uqlidisi considered  
zero a placeholder,  
not a number.**





**Al-Uqlidisi considered zero an empty placeholder, not a number.**

## ***Al-Uqlidisi (c. 920-980)***

One question is: **Why are the Hindi letters nine**, no more, no less? We say: Because the beginning of numbers from which they start is one and the last unit we pronounce is nine. Thus when we say units we mean something between one and nine; after that units are over, and ten comes out like one and takes its form. We add up ten to ten until we reach 90 which conforms with nine. Tens are now over and we say one hundred, coming back to one, and going up to 9. Thus we see that all places start with one and end with nine. That is why they are made nine. **So much for the nine letters.** If it is said: Why is zero multiplied by zero equal to zero and zero multiplied by any letter zero? We say that by multiplying zero by **zero the aim is only to occupy the place**; the same applies for multiplying the letter by zero. **We multiply the letter by zero only once, the first time, by the first letter in the upper, to occupy the place, and tell that there is a place and that it is empty.**

Saidan, Ahmad S. (1978) *The Arithmetic of Al-Uqlídisí: The Story of Hindu-Arabic Arithmetic As Told in Kitab Al-Fusul Fi Al-Hisab Al-Hindi*. Reidel, Dordrecht. P. 186





200 years after Brahmagupta,  
al-Khwārizmī did not accept 1 as a  
number. Zero as a number? Never!





200 years after Brahmagupta,  
al-Khwārizmī did not accept 1 as a  
number. Zero as a number? Never!



300 years after Brahmagupta,  
al-Uqlīdisī accepted India's **ZERO** as a  
placeholder, yet not a number. Why?





Al-Uqlīdisī means 'the Euclidist'. He was known for his skill in studying the Greek geometry of Euclid and translating it into Arabic.





Al-Uqlīdisī means 'the Euclidist'. He was known for his skill in studying the Greek geometry of Euclid and translating it into Arabic.

Around 300 BCE, Euclid defined 'number' as *a multitude of units*. So Euclid's definition of number came before 0 and 1 were numbers.



India defined zero as the sum of opposing negative and positive numbers or quantities with the same multitude or magnitude.



India defined zero as the sum of opposing negative and positive numbers / quantities with the same multitude or magnitude.

If Arabic and European writers in medieval times *really* understood India's zero, where are all the negative numbers in their writings?



*“I have read a few dozen medieval Arabic books on arithmetic and algebra, and there is no hint of negative numbers in any of them. Zero, too, was not regarded to be a number, but was merely the place holder for an empty place in the representation of a number in Arabic (Indian) notation.”*

By email courtesy of Dr. Jeffrey Oaks, Professor of Mathematics  
Medieval Arabic algebra and the mathematics of Greece and medieval Europe UINDY



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*“All numbers in Arabic arithmetic were positive. No Arabic author to my knowledge ever even contemplated the existence of negative numbers.”*

**By email courtesy of Dr. Jeffrey Oaks, Professor of Mathematics  
Medieval Arabic algebra and the mathematics of Greece and medieval Europe UINDY**



# The maths MYTHS we know

India's definition of zero as a number that was a sum of equal, yet opposite negative and positive quantities was **[NOT]** embraced by the Arabic world



# The maths MYTHS we know

India's definition of zero as a number that was a sum of equal, yet opposite negative and positive quantities was [NOT] embraced by the Arabic world

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which **[DID NOT]** feature Brahmagupta's ancient laws of sign for negatives and positives



# The maths MYTHS we know

Based on what he learned and documented in his book *Algoritmi de numero Indorum* (al-Khwārizmī on the Hindu Art of Reckoning) he then wrote a book on algebra **[NO]**

Al-Khwārizmī wrote his book on algebra BEFORE he understood Indian mathematics.



# The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.



# The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

Al-Khwārizmī simply eliminated any positive term that was being subtracted in an equation.



# The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

Al-Khwārizmī simply eliminated any positive term that was being subtracted in an equation.

For example,  $ax^2 = bx - c$  became  $ax^2 + c = bx$ .



# The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's,

Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.



# The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's

Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

“Once again al-Khwārizmī differs from Brahmagupta, this time in not employing any abbreviation.



# The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's

Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

Al-Khwārizmī... avoids using “negative” numbers or simply a [larger] number subtracted from a smaller one, or from zero, whereas Brahmagupta, like other Indian mathematicians before him, does not hesitate to make use of such [negative] numbers.”



# The maths MYTHS we know

“It is difficult to imagine that al-Khwārizmī, if he had read this chapter [i.e. chapter 18 of Brahmagupta’s Brāhma Sphutasiddhānta] would not have been able to profit by it, even if only to shorten the presentation of his work.”



# The maths MYTHS we know

“It is difficult to imagine that al-Khwārizmī, if he had read this chapter [i.e. chapter 18 of Brahmagupta’s Brāhma Sphutasiddhānta] would not have been able to profit by it, even if only to shorten the presentation of his work.”

“The style of the mathematical reasoning that is at work in al-Khwārizmī’s algebra has nothing to do with what we encounter in the work of his (Indian) predecessors.”



**628 CE**  
**Brahmagupta**  
**had everything**  
**we need today!**





**628 CE**  
**Brahmagupta**  
**had everything**  
**we need today!**



**830 CE**  
**Al-Khwārizmī**  
**did not have 1**  
**as a number.**





**628 CE**  
**Brahmagupta**  
**had everything**  
**we need today!**



**830 CE**  
**Al-Khwārizmī**  
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**950 CE**  
**Al-Uqlīdisī**  
**only had 0 as**  
**a placeholder**



**628 CE**  
**Brahmagupta**  
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**we need today!**

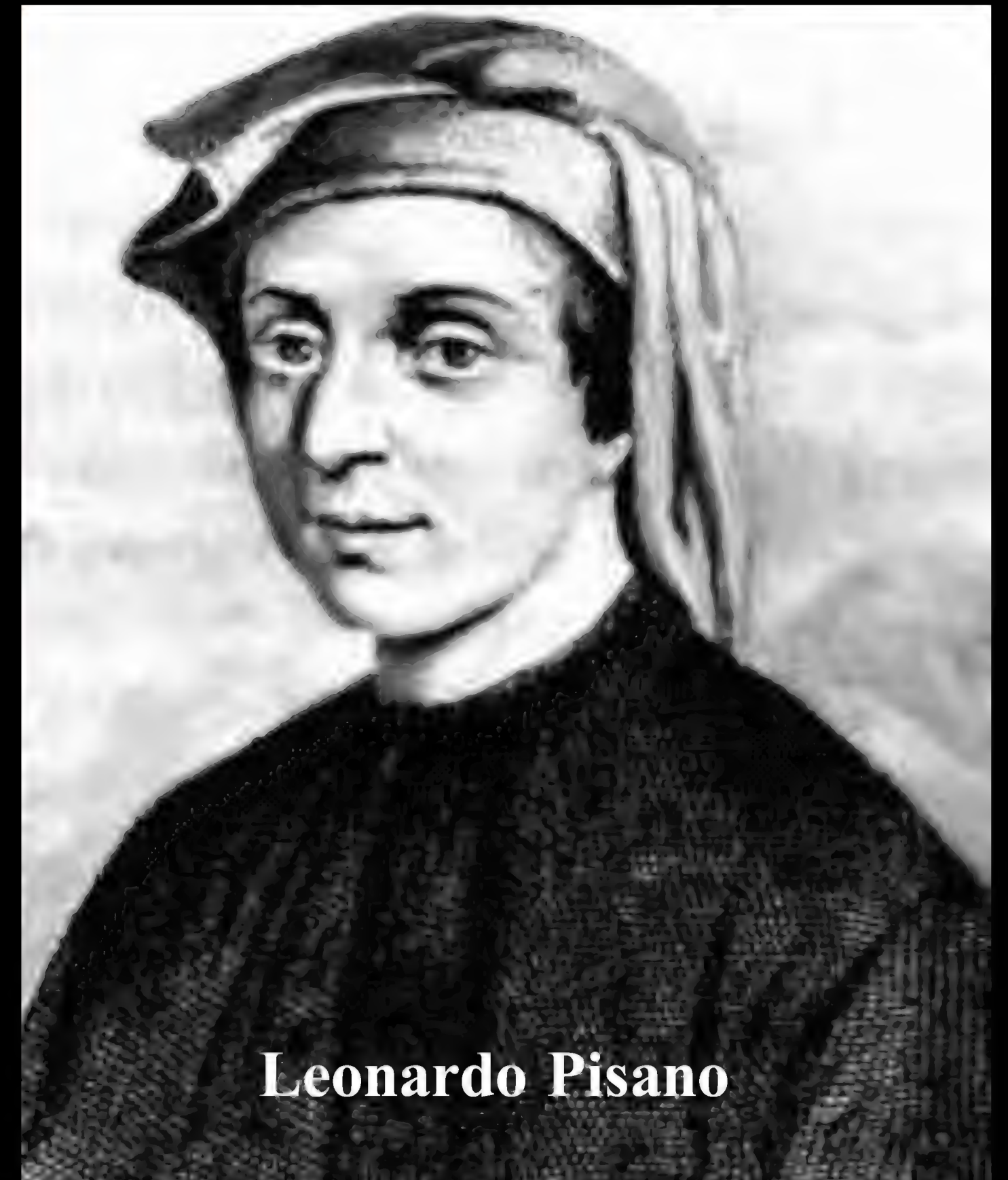


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**Leonardo Pisano**  
**1170 – 1250**



**Leonardo Pisano**

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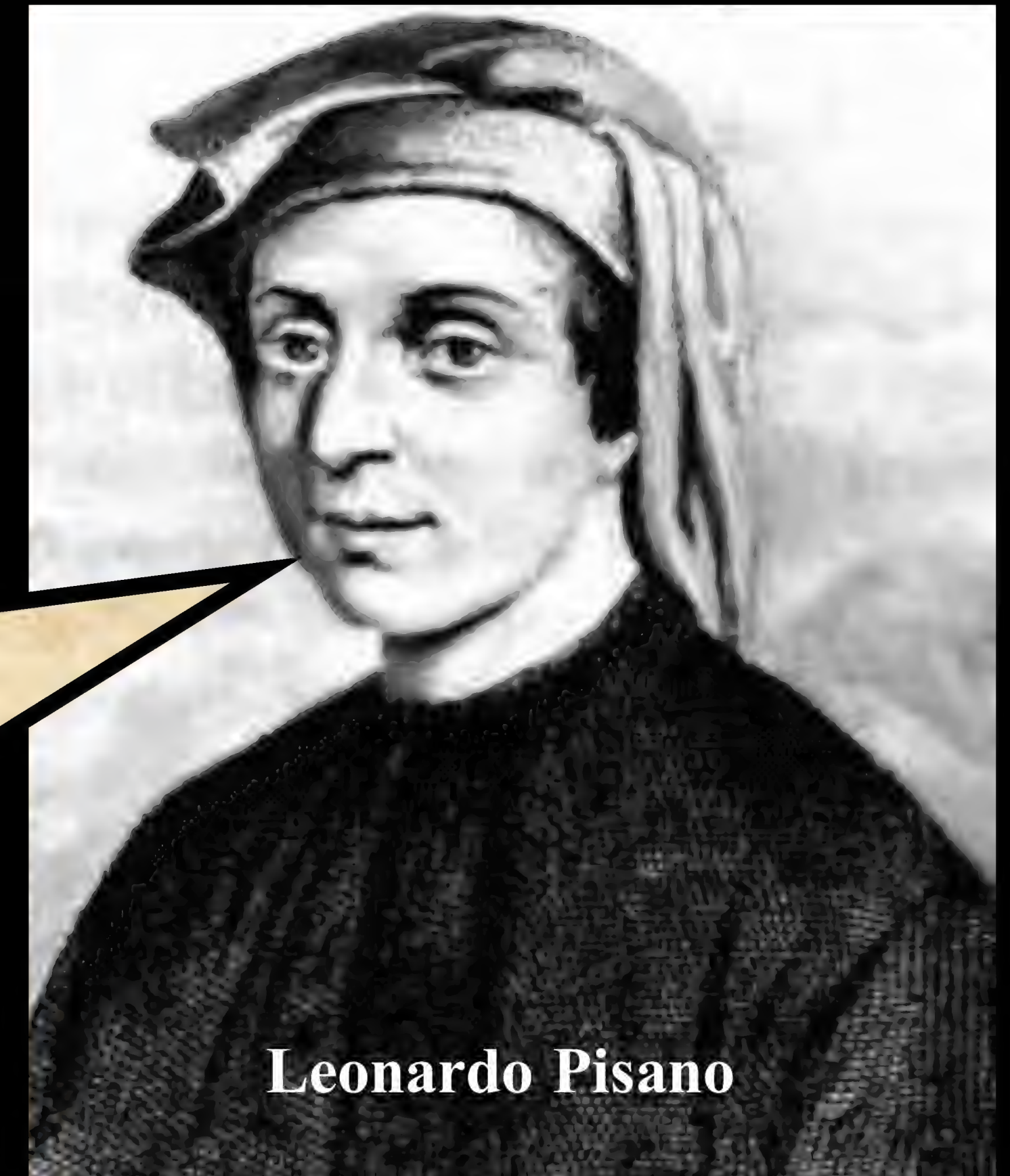


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950 CE  
Al-Uqlīdisī  
only had 0 as  
a placeholder

**I am Leonardo Pisano. I am the  
man most responsible for  
introducing India's arithmetic  
into Europe in the 13<sup>th</sup> Century  
via my book Liber Abaci.**



**Leonardo Pisano**



**628 CE**  
**Brahmagupta**  
**had everything**  
**we need today!**

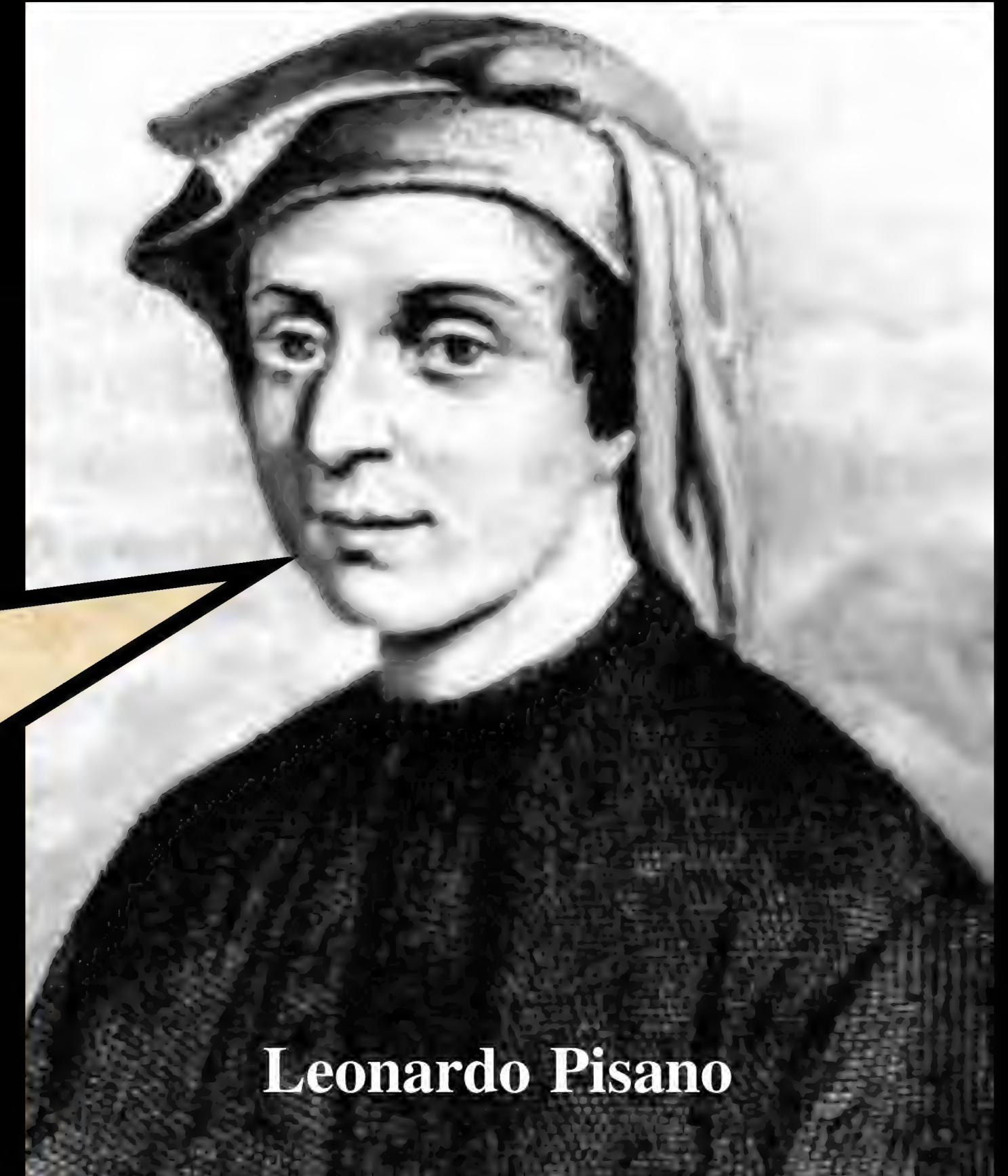


830CE  
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950 CE  
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As I got my Indian info via  
Arabic traders, I did **NOT** get to  
learn about India's definition of  
zero as a number or the rules of  
positive and negatives.



Leonardo Pisano



**628 CE**  
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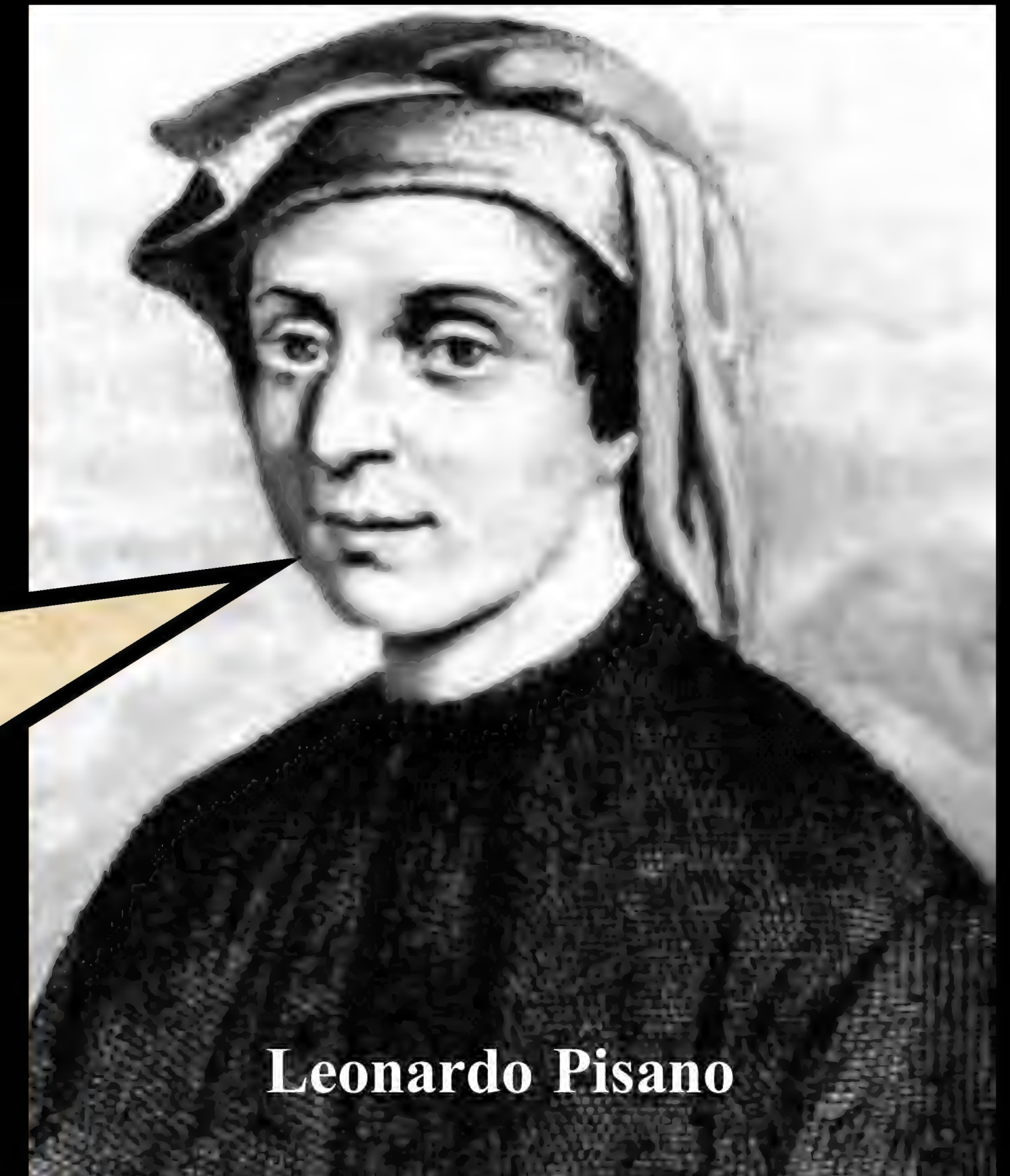


830CE  
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as a number.



950 CE  
Al-Uqlīdisī  
only had 0 as  
a placeholder

As I got my Indian info via  
Arabic traders, I did **NOT** get to  
learn about India's definition of  
zero as a number or the rules of  
positive and negatives. **Whoops!**



Leonardo Pisano

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**[www.j.mp/IndiasMaths](http://www.j.mp/IndiasMaths)**





Transmission of zero  
as a placeholder, yet  
not as defined by  
Brahmagupta 628 CE.







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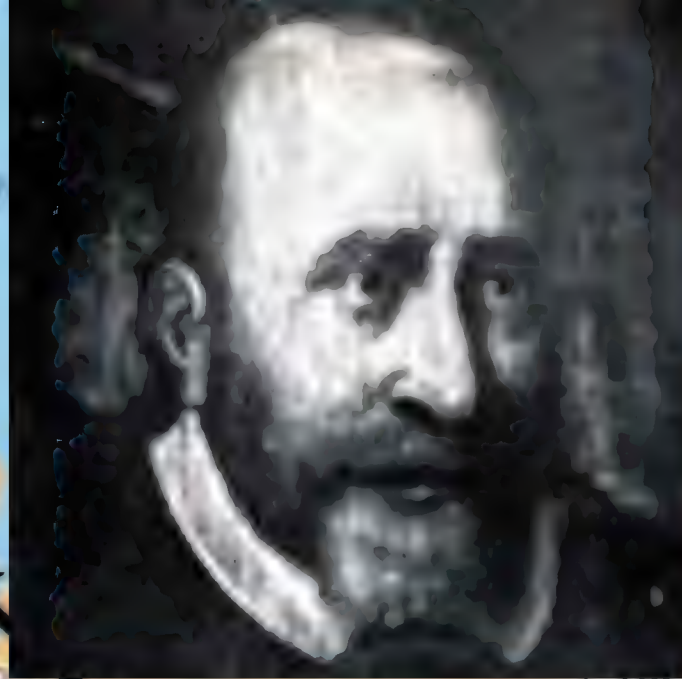




Transmission of zero  
as a placeholder, yet  
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Transmission of zero  
as a placeholder, yet  
not as defined by  
Brahmagupta 628 CE.

Those unaware zero was defined  
as the sum of equal pos. & neg.

- Al-Khwārizmī, Iraq 9th C.
- Traders, Nth Africa 12th C.
- Leonardo Pisano, Italy 13th C.
- Robert Recorde, England 16th C.



I invented the sign = and introduced the pre-existing sign + to England.



Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

Those unaware zero was defined as the sum of equal pos. & neg.

- **Al-Khwārizmī, Iraq 9th C.**
- **Traders, Nth Africa 12th C.**
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Map Data © 2019 Google



Yet I never knew about your zero definition Mr Brahmagupta or laws of positives & negatives

Recordes did know subtraction subtracted is addition yet that's a different idea.



Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

Those unaware zero was defined as the sum of equal pos. & neg.

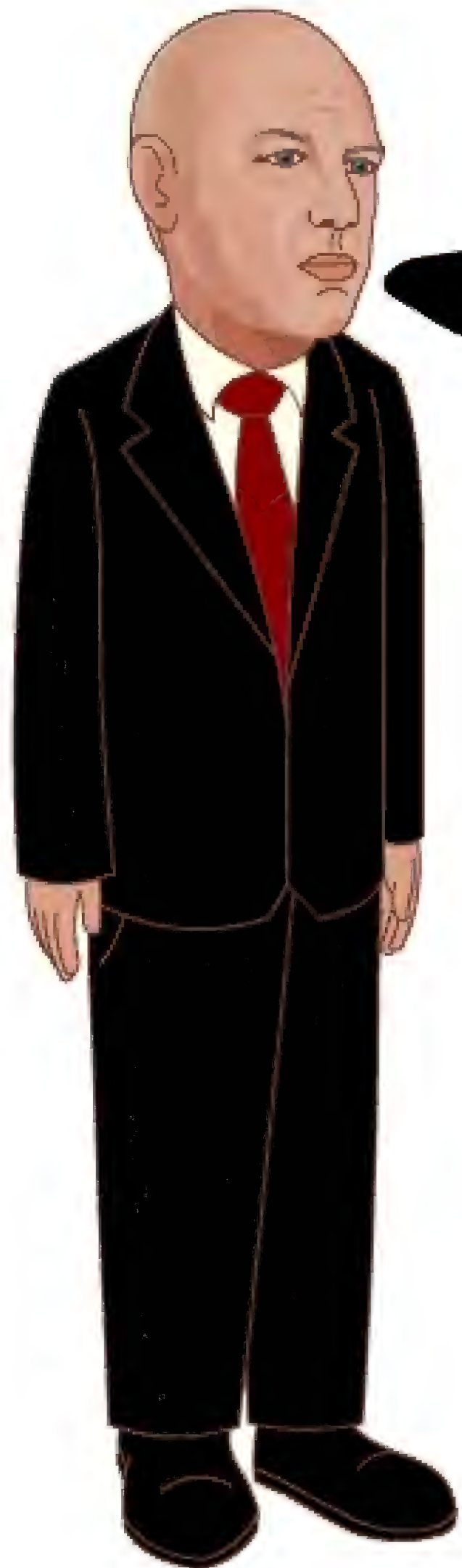
- Al-Khwārizmī, Iraq 9th C.
- Traders, Nth Africa 12th C.
- Leonardo Pisano, Italy 13th C.
- Robert Recorde, England 16th C.

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Map Data © 2019 Google



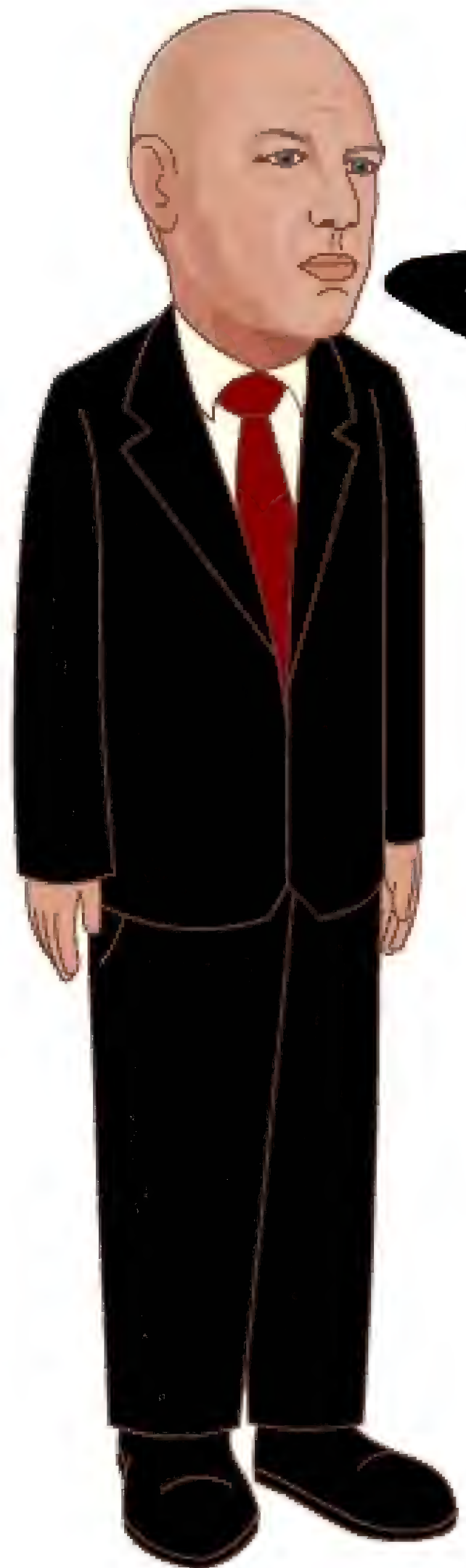


**In 1478 the first book printed  
on maths (Treviso Arithmetic)  
said numbers start at 2.**

**So much for 0 and 1 which is  
all your computer needs!**

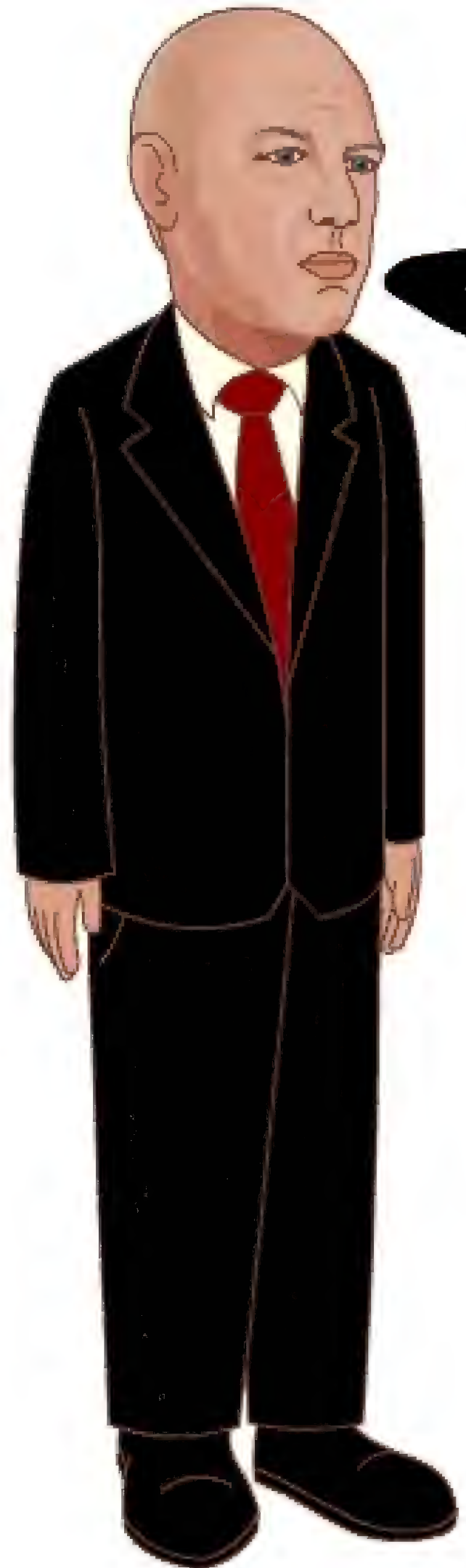
**[www.j.mp/IndiasMaths](http://www.j.mp/IndiasMaths)**





**In 16<sup>th</sup> Century England  
people used Roman  
Numerals and there was  
no Roman Numeral for 0.**





**The maths at the time  
was based on Ancient  
Greek maths which did  
not have zero, one or  
negative numbers.**



The false idea negative quantities are less than zero (rather than opposite in nature to positive quantities) emerged in Michael Stiefel's *Arithmetica Integra* of 1544 in a section titled De signis additorum & subtractorum & de numeris absurdis.





**Michael Stiefel said negative numbers were below zero which is below nothing *infra 0, id est infra nihil* and negative numbers were absurd *numeri absurdi*.**





**To make sense of numbers that count or measure negative quantities, (i.e. negative numbers) all we need to do is drop the nonsense notion that negative quantities are 'less than zero'.**





**Then negative numbers  
simply count or measure  
opposite quantities or  
forces, which are always  
greater than zero!**





**Perhaps you might recall  
Newton's Third Law which  
states for every action there  
is an equal and opposite  
reaction. Bingo!**





**Newton's Laws of Motion  
are consistent with  
Brahmagupta's laws of  
quantitative mathematics,  
which are also consistent  
with quantum physics.**

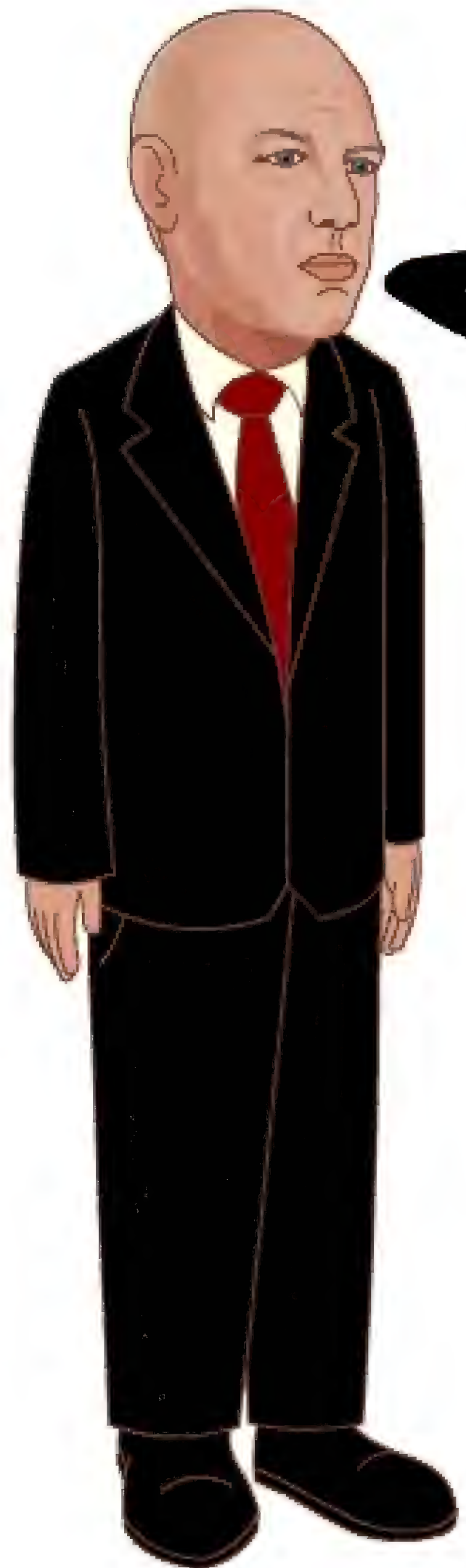




**Think about it... 3 negative  
electrons and 3 positive  
positrons cancel each  
other out to sum to zero.**

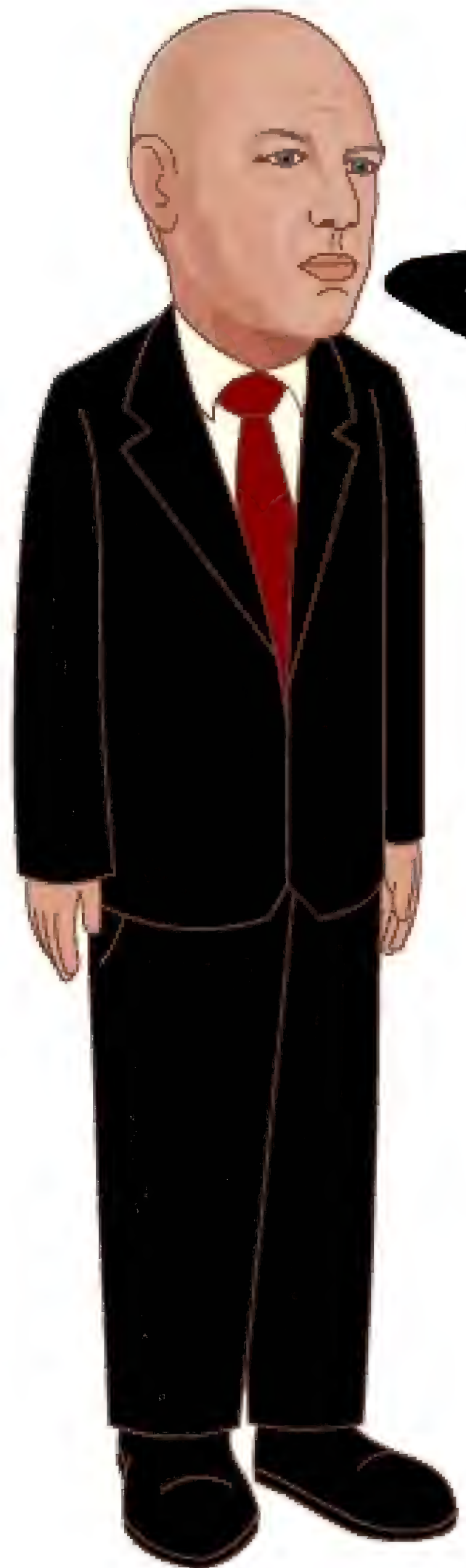






**As maths books got published in the English language, (without 0 or 1 in algorithmic definitions or as numbers) they were exported to England's settlements and colonies (e.g. New England became America).**





So as the English language spread, so too did **major misunderstandings** of India's mathematical foundations!



In 628 Brahmagupta gave  
solutions to equations we'd  
write today as  $x^2 - 92y^2 = 1^*$   
and  $ax^2 + bx + c = 0$ .





In 628 Brahmagupta gave  
solutions to equations we'd  
write today as  $x^2 - 92y^2 = 1^*$   
( $x = 1151$  and  $y = 120$ )





**However, the first person to say 1 was a number in the West was Simon Stevin in 1585, almost 1000 years after Brahmagupta!**





However, the first person to say 1 was a number in the West was Simon Stevin in 1585, almost 1000 years after Brahmagupta! **So, where is zero today?**







What FACTS do we know?

# Jonathan J. Crabtree

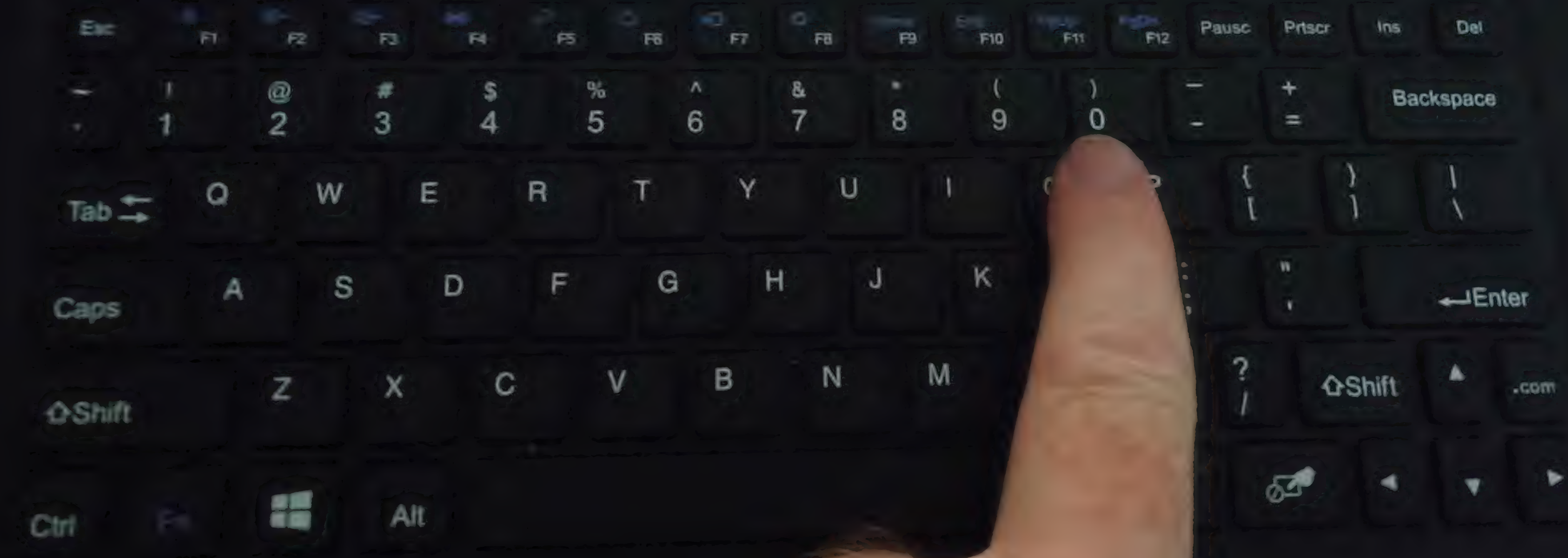
## Patna Science College, Patna University, Bihar India

Elementary Mathematics Historian, Melbourne, Australia

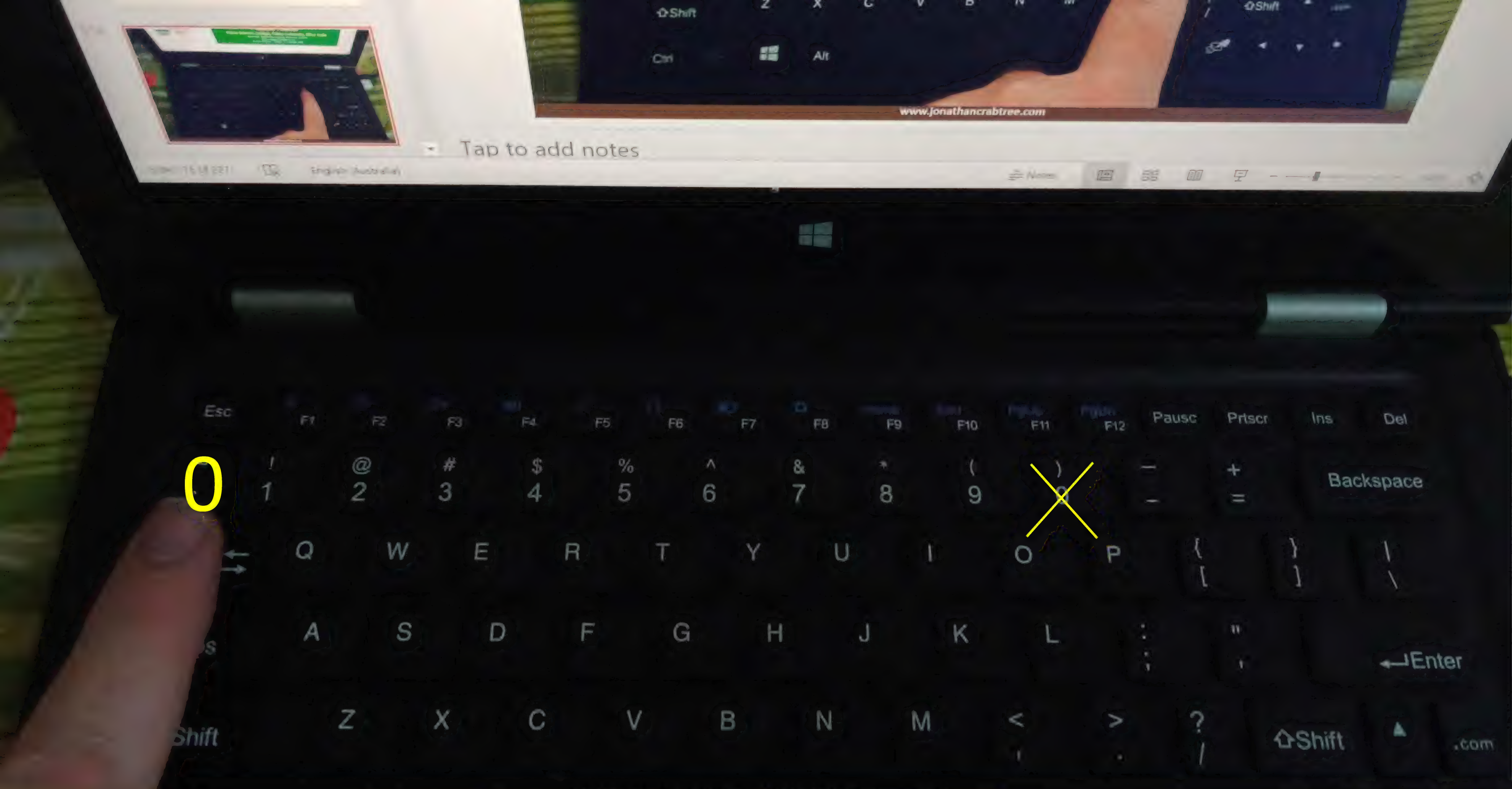
research@jonathancrabtree.com

© 2019 Jonathan J. Crabtree, 12 December, 2019.

Tap to add notes









# Brahmagupta's 5 Addition Laws

**AL1** positive plus positive is positive

**AL2** negative plus negative is negative

**AL3** positive plus negative is the difference between the positive and negative

**AL4** when positive and negative are equal the sum is zero

positive plus zero is positive

**AL5** negative plus zero is negative

zero plus zero is zero



# Brahmagupta's 5 Subtraction Laws

**SL1** A smaller **positive** subtracted from a larger **positive** is **positive**.

**SL2** A smaller **negative** subtracted from a larger **negative** is **negative**.

**SL3** If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative** becomes **positive** and **positive** **negative**.

**SL4** A **negative** minus **zero** is **negative**,  
a **positive** minus **zero** is **positive**,  
**zero** minus **zero** is **zero**.

**SL5** When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.



# Brahmagupta's 4 Multiplication Laws

**ML1** The product of a **negative** and a **positive** is **negative**.

**ML2** The product of two **negatives** is **positive**.

**ML3** The product of two **positives** is **positive**.

**ML4** The product of **zero** and a **negative**,  
of **zero** and a **positive**, or  
of two **zeros** is **zero**.



# Brahmagupta's 4 Division Laws

**DL1** A **positive** divided by a **positive** is **positive**.

**DL2** A **negative** divided by a **negative** is **positive**.

**DL3** A **positive** divided by a **negative** is **negative**.

**DL4** A **negative** divided by a **positive** is **negative**.

Acknowledgement: I am grateful to Avinash Sathaye, K. Ramasubramanian, Clemency Montelle, Kim Plofker and Agathe Keller. Analysis, interpretation (*& any mistakes*) by Jonathan J. Crabtree.



# Brahmagupta's 5 Addition Laws

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**AL5** negative plus zero is negative

zero plus zero is zero

# Brahmagupta's 4 Multiplication Laws

**ML1** The product of a negative and a positive is negative.

**ML2** The product of two negatives is positive.

**ML3** The product of two positives is positive.

**ML4** The product of zero and a negative,  
of zero and a positive, or  
of two zeros is zero.

# Brahmagupta's 5 Subtraction Laws

**SL1** A smaller positive subtracted from a larger positive is positive.

**SL2** A smaller negative subtracted from a larger negative is negative.

**SL3** If a larger negative or positive is to be subtracted from a smaller negative or positive, the sign of their difference is reversed – negative becomes positive and positive negative.

**SL4** A negative minus zero is negative,  
a positive minus zero is positive,  
zero minus zero is zero.

**SL5** When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

# Brahmagupta's 4 Division Laws

**DL1** A positive divided by a positive is positive.

**DL2** A negative divided by a negative is positive.

**DL3** A positive divided by a negative is negative.

**DL4** A negative divided by a positive is negative.



When **positive** and **negative** are equal the sum is **zero**





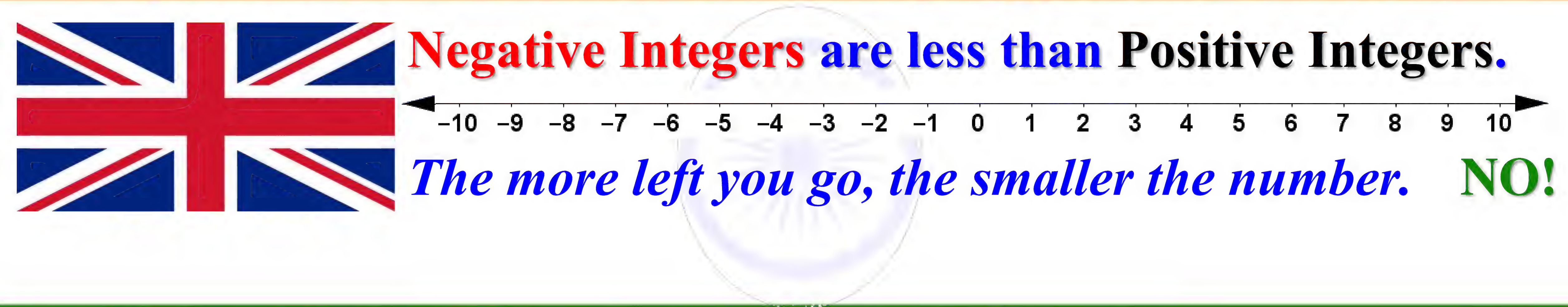
When **positive** and **negative** are equal the sum is **zero**  
Correct according to the laws of physics,  
yet **NOT** what we are taught in school!





When **positive** and **negative** are equal the sum is **zero**

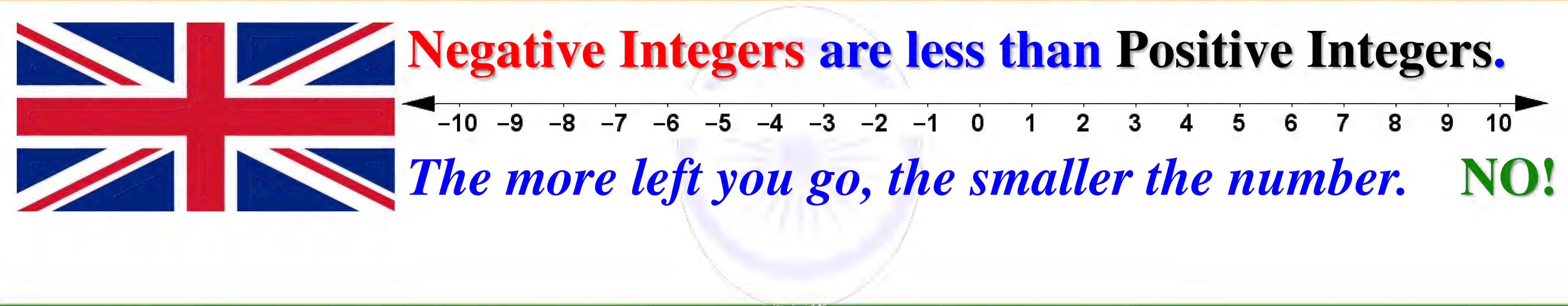
Correct according to the laws of physics,  
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When **positive** and **negative** are equal the sum is **zero**

Correct according to the laws of physics,  
yet **NOT** what we are taught in school!



Maths pedagogies were developed on **Naturals  $\mathbb{N}$**   
then extended via subtraction to form **Integers  $\mathbb{Z}$**



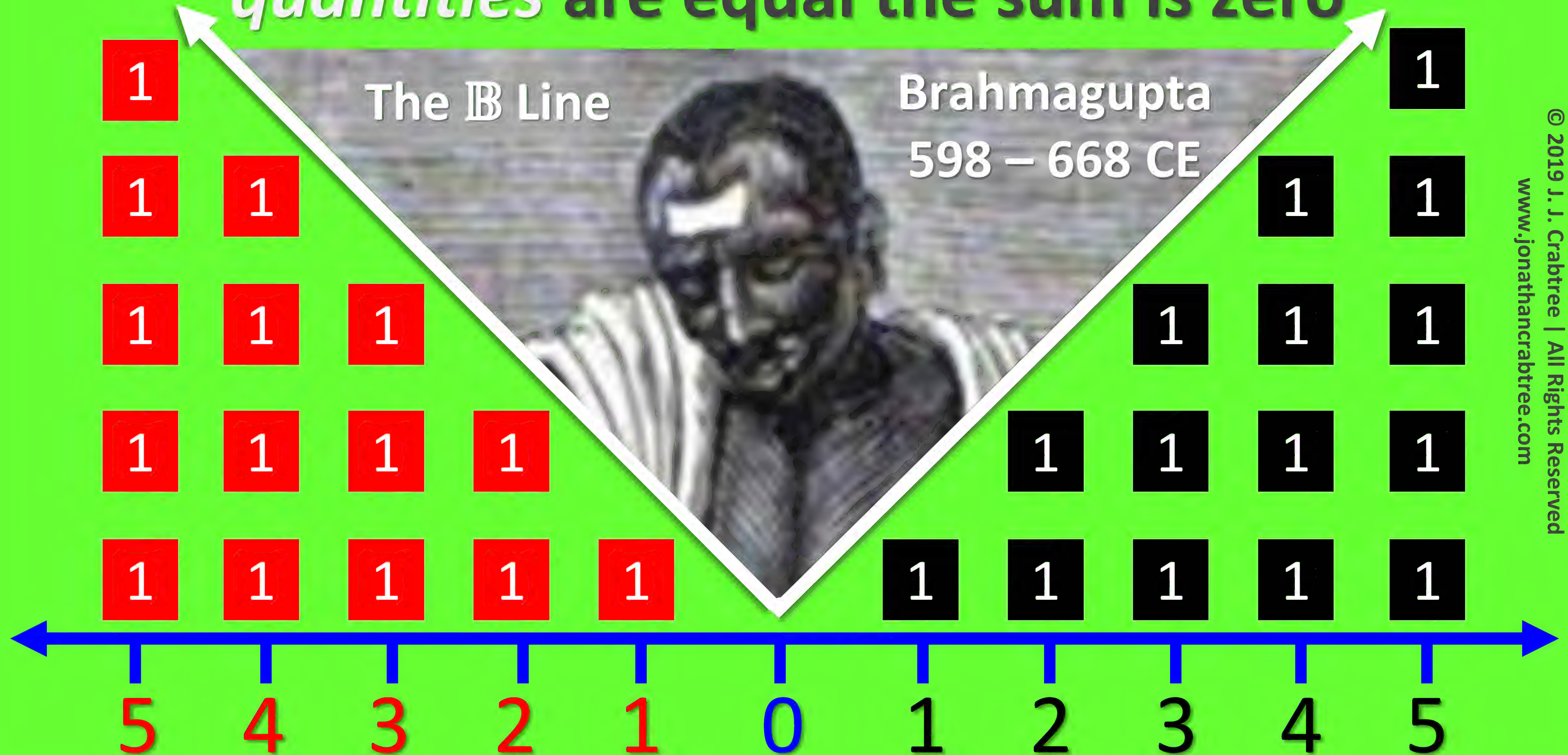
1	1	1	1	1	minus	1		+ 4 equals	1	1	1	1
1	1	1	1	1	minus	1	1	+ 3 equals	1	1	1	
1	1	1	1	1	minus	1	1	1	+ 2 equals	1	1	
1	1	1	1	1	minus	1	1	1	1	+ 1 equals	1	
1	1	1	1	1	minus	1	1	1	1	1	0 equals	<b>N</b>

1	1	1	1	1	minus	1	1	1	1	1	1	- 1 equals	1			
1	1	1	1	1	minus	1	1	1	1	1	1	1	- 2 equals	1	1	<b>Z</b>

**The West wrongly extended the Naturals pattern onto the Integers.**



***“when the number of positive and negative quantities are equal the sum is zero”***





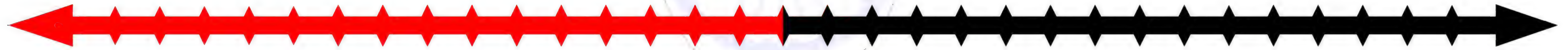
# Seeing Simple Symmetries of Quantity

**Counts or measures  
of negative units**

**Counts or measures  
of positive units**

**Electrons South West Left Down Debts Loss  
Deaths Emigration Cold Decay Below Zero  
Less Than Enough Below Ground To the hour  
Deceleration Head Wind (knots) Under Par (golf)**

**Positrons North East Right Up Assets Profit  
Births Immigration Heat Growth Above Zero  
More Than Enough Above Ground Past the hour  
Acceleration Tail Wind (knots) Over Par (golf)**





# Brahmagupta's 5 Subtraction Laws

**SL1** A smaller **positive** subtracted from a larger **positive** is **positive**.

$$+9 - +2 = +7$$

**SL2** A smaller **negative** subtracted from a larger **negative** is **negative**.

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$$+4 - +6 = -2$$

**SL4** A **negative** minus **zero** is **negative**,  
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$$+4 - +6 = -2$$

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# Brahmagupta's 5 Subtraction Laws

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or a **negative** from a **positive**, then it is to be added.

Smaller  
Negative!


$$-8 - (-5) = -3$$



# Brahmagupta's 5 Subtraction Laws

**SL1** A smaller **positive** subtracted from a larger **positive** is **positive**.

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or a **negative** from a **positive**, then it is to be added.

Smaller  
Negative!

$$-8 - (-5) = -3$$

$$-3 - (-7) = +4$$

Larger  
Negative!



# Brahmagupta

**SL1** A smaller

**SL2** A small

**SL3** If a large  
negative  
negative

**SL4** A negative  
a positive  
zero

**SL5** When a positive  
or a negative

## on Laws

$$-8 - (-5) = -3$$

Smaller  
Negative!

$$-3 - (-7) = +4$$

Larger  
Negative!

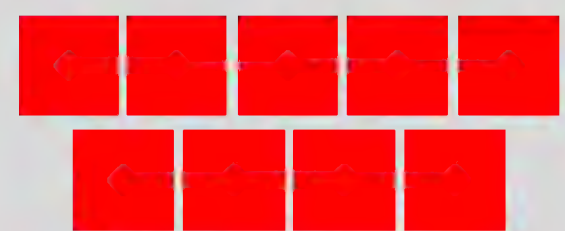
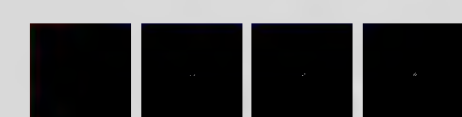




# Seeing Maths with Indian Eyes

Which numbers are **greater**?

 or 

 or 

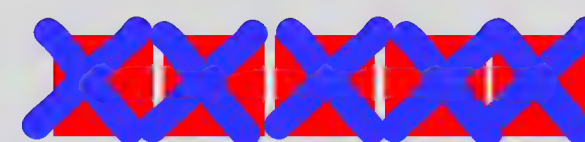
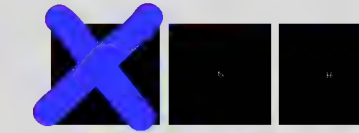
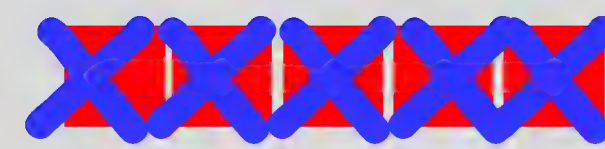
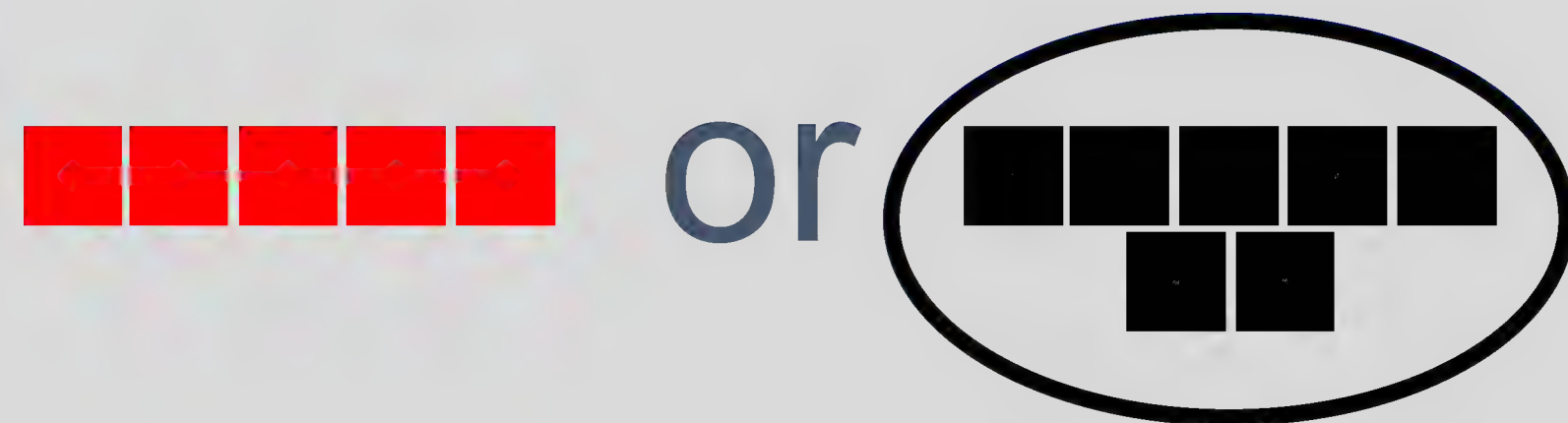
 or 

 or 



# Seeing Maths with Indian Eyes

Which numbers are **greater**?

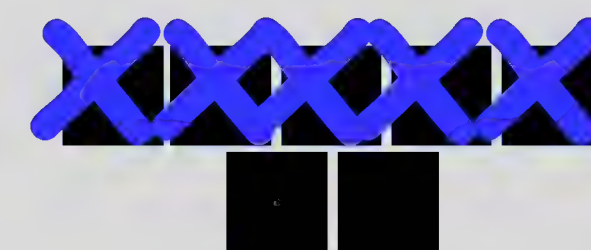
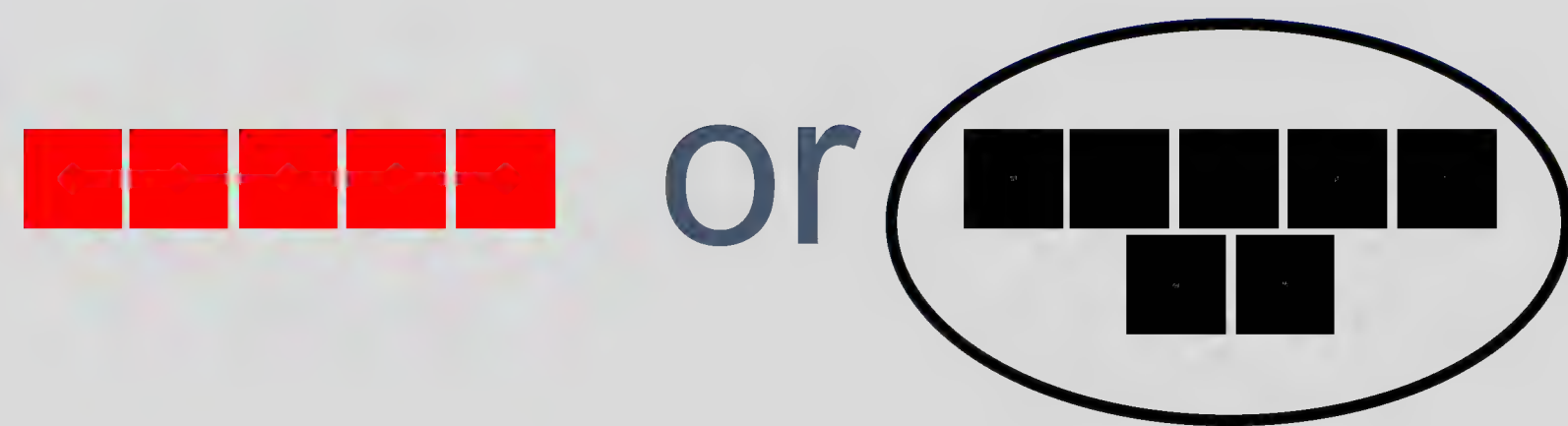




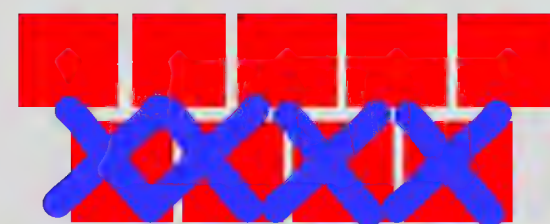
# Seeing Maths with Indian Eyes

Which numbers are **greater**?

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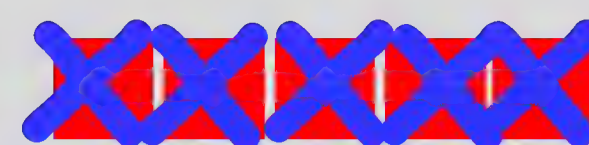
$$5^- < 7^+$$



$$9^- > 4^+$$

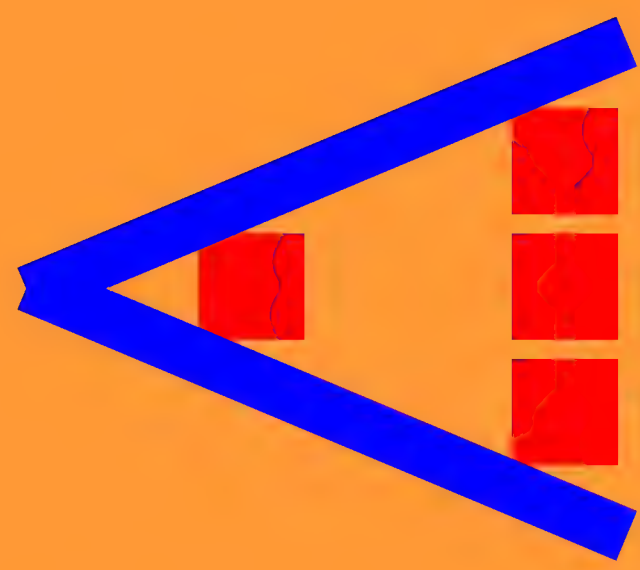


$$1^- < 3^+$$

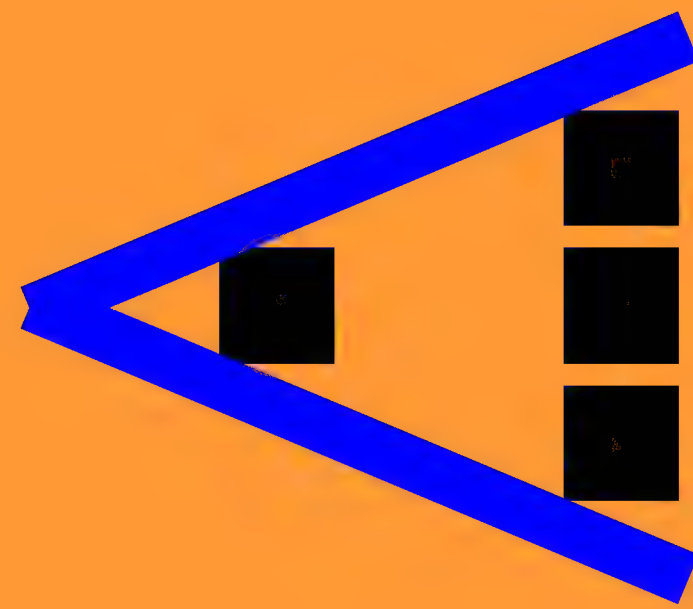


$$5^- \not= 5^+$$

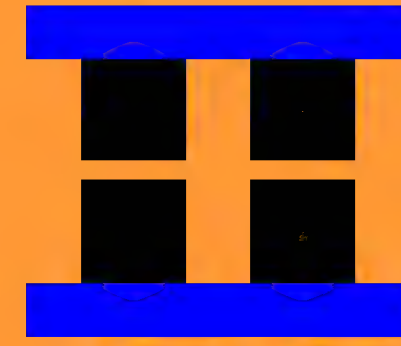




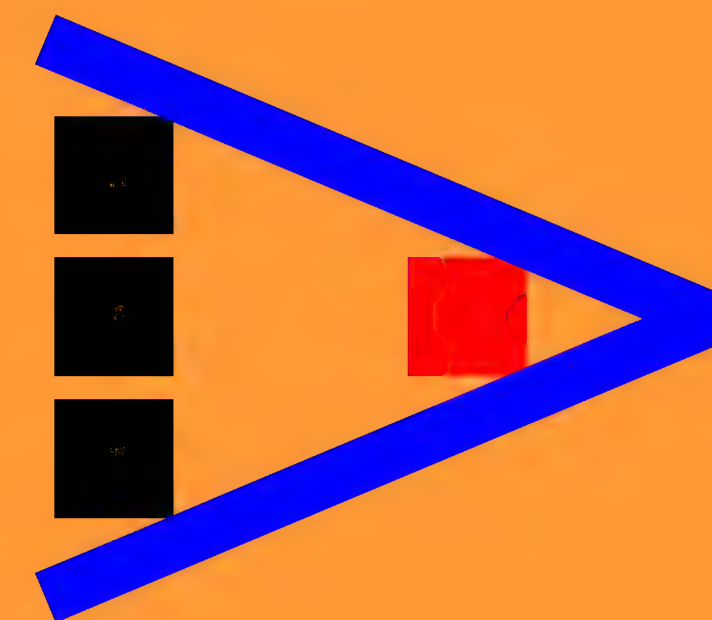
$$1^- < 3^-$$



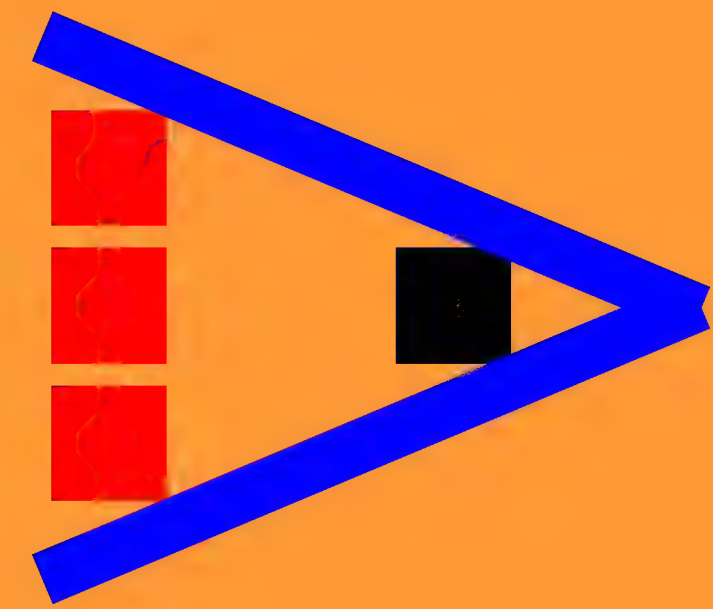
$$1^+ < 3^+$$



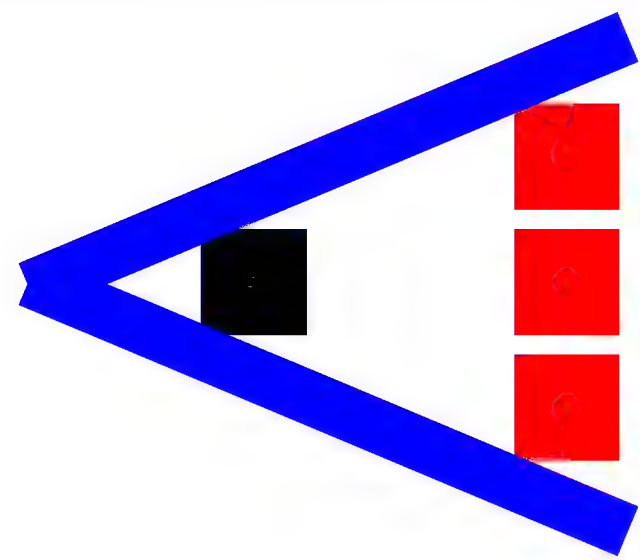
$$2^+ = 2^+$$



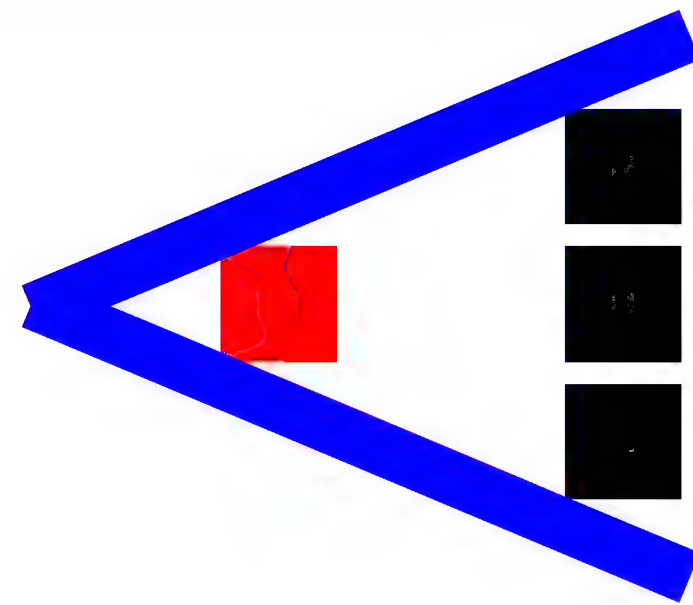
$$3^+ > 1^-$$



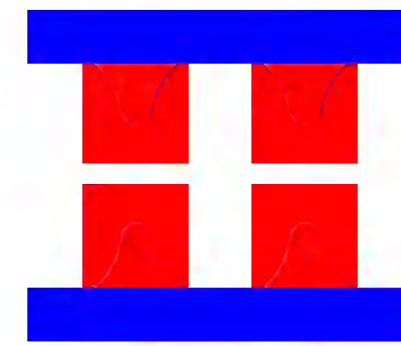
$$3^- > 1^+$$



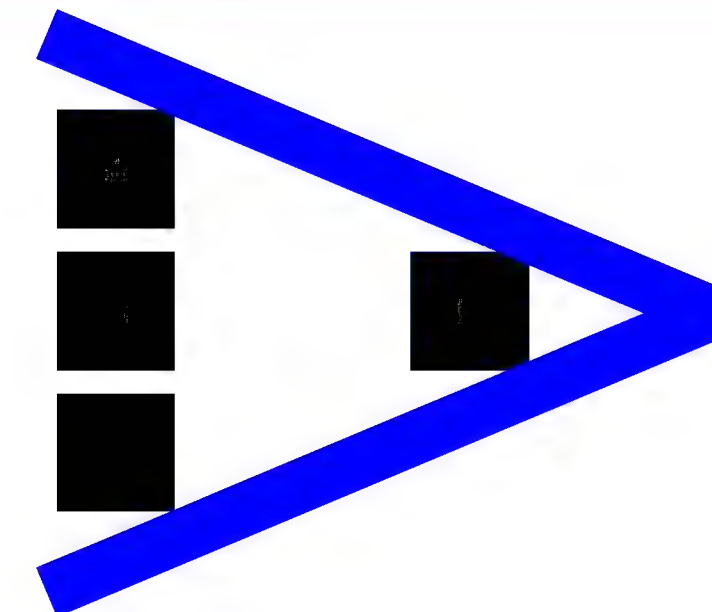
$$1^+ < 3^-$$



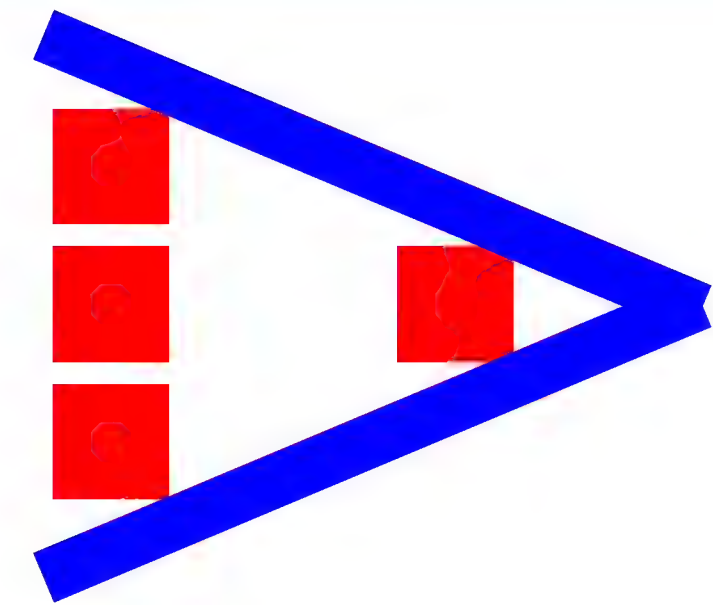
$$1^- < 3^+$$



$$2^- = 2^-$$



$$3^+ > 1^+$$



$$3^- > 1^-$$




$2^-$  are equal & opposite to  $2^+$

$$2^- \ominus 2^+$$

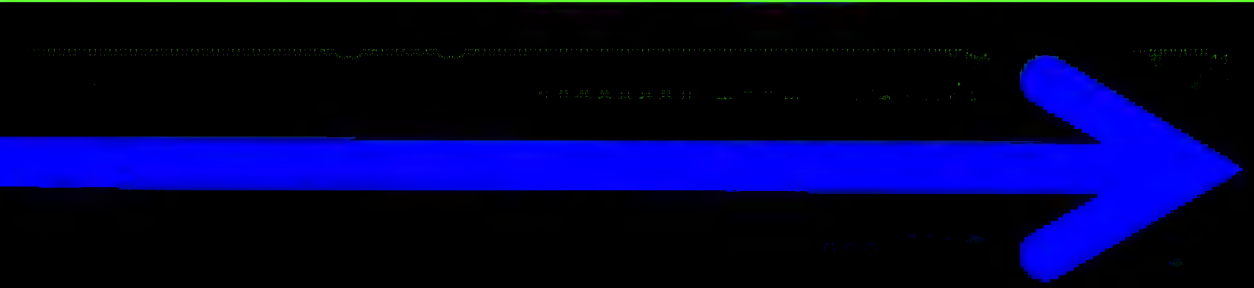


# JJC ASSESSMENT OF THE WORLD'S PEDAGOGICAL EVOLUTION (628 to Now)

$+12 + +4$	$+12 + -4$	$-12 + +4$	$-12 + -4$
$+12 - +4$	$+12 - -4$	$-12 - +4$	$-12 - -4$
$+12 \times +4$	$+12 \times -4$	$-12 \times +4$	$-12 \times -4$
$+12 \div +4$	$+12 \div -4$	$-12 \div +4$	$-12 \div -4$
	<b>PASS</b>	<b>FAIL</b>	<b>Absent</b>



# Podometric™ set to replace Arithmetic Dec. 2020

$+12 + +4$	$+12 + -4$	$-12 + +4$	$-12 + -4$
$+12 - +4$	$+12 - -4$	$-12 - +4$	$-12 - -4$
$+12 \times +4$	$+12 \times -4$	$-12 \times +4$	$-12 \times -4$
$+12 \div +4$	$+12 \div -4$	$-12 \div +4$	$-12 \div -4$
	PASS	FAIL	Absent



# Podometric™ set to replace Arithmetic Dec. 2020 with Free maths eBooks for every Indian child

$$+12 + +4$$

$$+12 - +4$$

$$+12 \times +4$$

$$+12 \div +4$$

$$+12 + -4$$

$$+12 - -4$$

$$+12 \times -4$$

$$+12 \div -4$$

$$-12 + +4$$

$$-12 - +4$$

$$-12 \times +4$$

$$-12 \div +4$$

$$-12 + -4$$

$$-12 - -4$$

$$-12 \times -4$$

$$-12 \div -4$$



**PASS**



**Hey kids, play with me  
and I'll make maths fun  
fast and free to learn!**





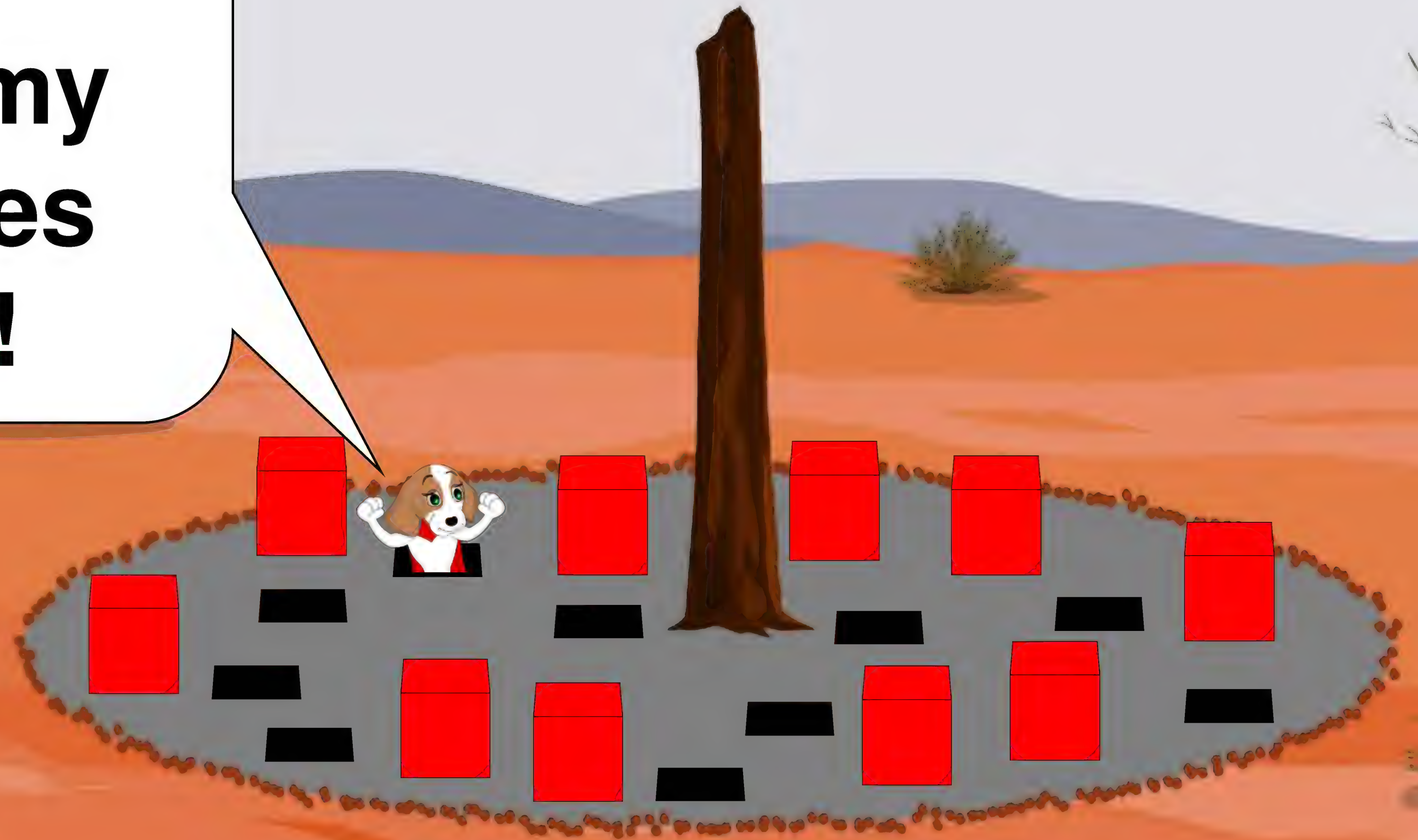
Hey kids, play with me  
and I'll make maths fun  
fast and free to learn!

**From Class 1 on, it all  
connects with the laws of  
physics as well!**





**Separate, or  
altogether, all my  
bricks and holes  
give me **zero**!**





**I'm Podo the Puppy.**  
**All my maths adventures**  
**are being brought to life by**  
**AFX Animation in Kolkata.**  
**[www.afxanimation.com](http://www.afxanimation.com)**





**The hard work has been done!**  
**India can update its**  
**maths and prosper**  
**or let this major maths**  
**education advantage pass by.**



**KEEP  
DELHI  
CLEAN**









# Big Problems Demand We



# Think bigger

## Local Legends



**ANDREW MATHIESON**

BURIED among the hundreds of everyday emails in Jonathan Crabtree's inbox are a few worth keeping.

Replies from Nelson Mandela, Bishop Desmond Tutu, the Dalai Lama, even Muhammad Ali are quickly printed out and put aside for safe keeping.

They are a reminder of the power behind the written word; that ordinary people can make a difference.

"I haven't got one from the President of the United States or the Pope yet," Jonathan says, "but I'll get there soon."

"The emails are not really that important - it's much more about the ideas."

The world is not his enemy, for Jonathan's work



**Brain power: Jonathan Crabtree has endless ideas for making the world a better place.**

*Picture: Tommy Ritchie*

truck trying to beat a red light 25 years ago nearly left him dead on the road after colliding with his motorcycle.

Lying motionless on the ground, Jonathan could only remember the horizon spinning around as if he were in a plane going down.

Consequently, the driver lost his licence but the crash was forgiven for Jonathan when going to

despite no formal qualifications, took up teaching mathematics from home.

"I actually failed maths," he laughs.

Throwing away the classroom text books, Jonathan taught kids to imagine algebra by closing their eyes and listening to fairytales.

Education authorities in Australia first shunned





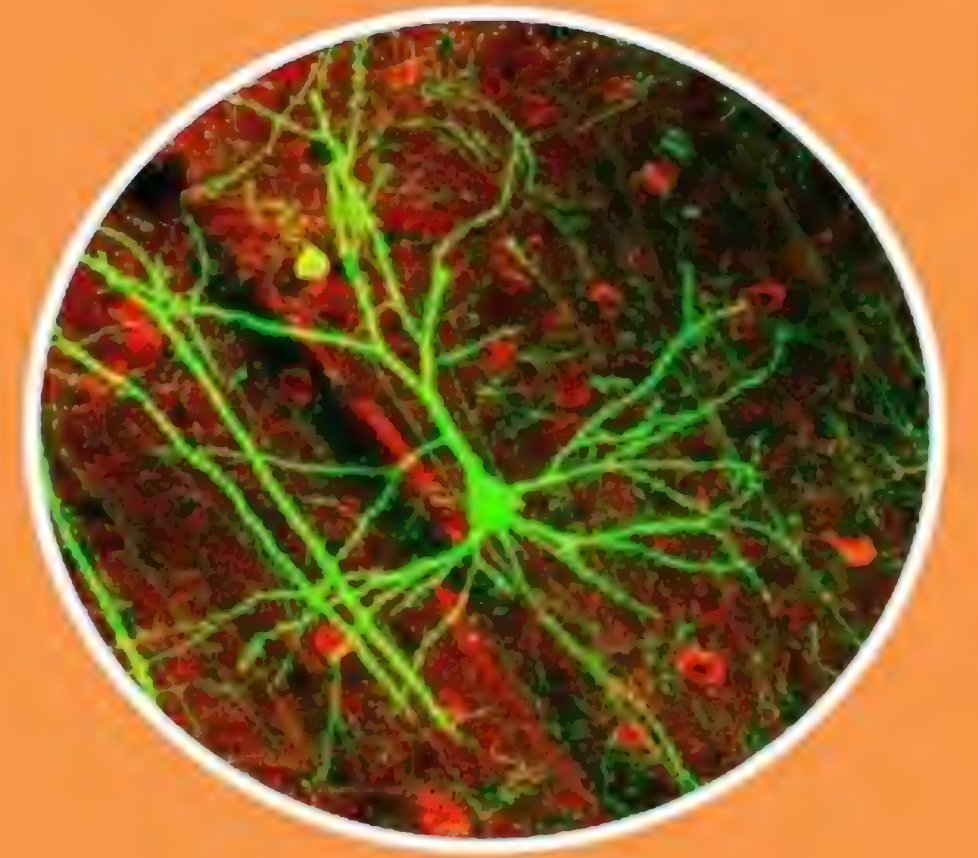
**Science**



**Technology**



**Engineering**



**Medicine**

In the West

**MATHEMATICS**



**India's teachers so seldom know  
The trees of knowledge from seeds they sow  
Past lives forgotten and the future a mystery  
Making lives count, their deeds have made history**

A Seed to a View by Jonathan J. Crabtree

**So make your life count with love as your measure  
Then kids will climb trees with views they will treasure.**



**Thank you!**

**If You Care  
Please Share**

**[www.j.mp/IndiasMaths](http://www.j.mp/IndiasMaths)**



***Thank You!***

***Please email me your feedback***  
**research@jonathancrabbtree.com**



**Jonathan J. Crabbtree**

Elementary Mathematics Historian, Melbourne, Australia

research@jonathancrabbtree.com

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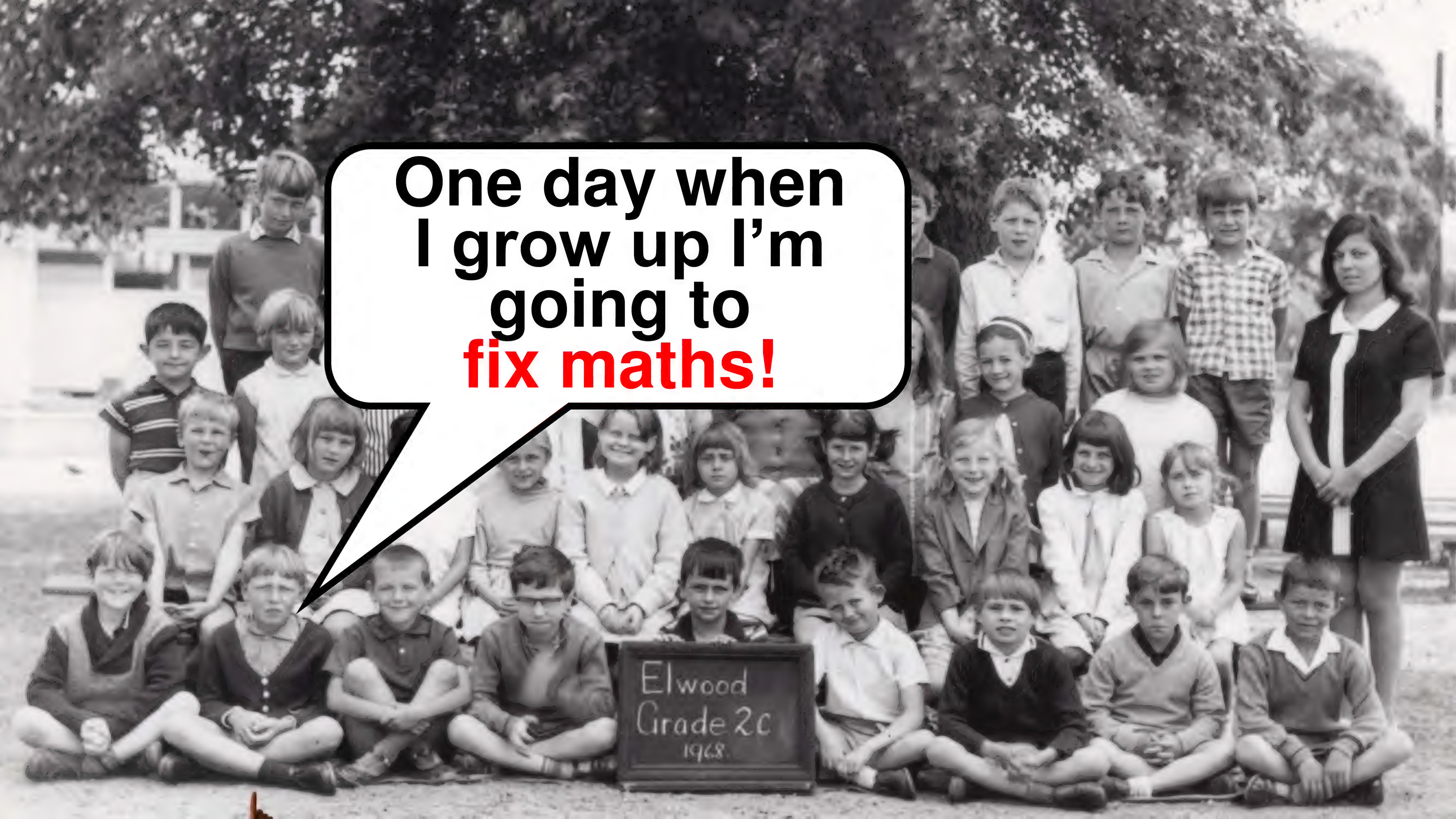




**I am Jonathan.  
I am seven. I like  
cartoons but,  
I don't like maths!**

Elwood  
Grade 2C  
1968





One day when  
I grow up I'm  
going to  
**fix maths!**

Elwood  
Grade 2C  
1968



# PISA Mathematics Survey?

(Programme for International Student Assessment)

In **Tamil Nadu** and **Himachal Pradesh** **15%** and **12%** of students are ready to use mathematics in ways that are considered fundamental for their future development.

<https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies>



# PISA Mathematics Survey?

(Programme for International Student Assessment)

In **Tamil Nadu** and **Himachal Pradesh** **15%** and **12%** of students are ready to use mathematics in ways that are considered fundamental for their future development.

The **OECD** average is **75%**.

<https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies>



# PISA Science Survey?

(Programme for International Student Assessment)

In **Tamil Nadu** and **Himachal Pradesh** **16%** and **11%** of students are proficient in science ... to participate actively in life situations related to science and technology.

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# PISA Science Survey?

(Programme for International Student Assessment)

In **Tamil Nadu** and **Himachal Pradesh** **16%** and **11%** of students are proficient in science ... to participate actively in life situations related to science and technology.

The **OECD** average is **82%**.

<https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies>



# The English support the English cricket team



**The English support the English cricket team**

**Indians support the Indian cricket team**



**The English support the English cricket team**

**Indians support the Indian cricket team**

**The English support English mathematicians**



**The English support the English cricket team**

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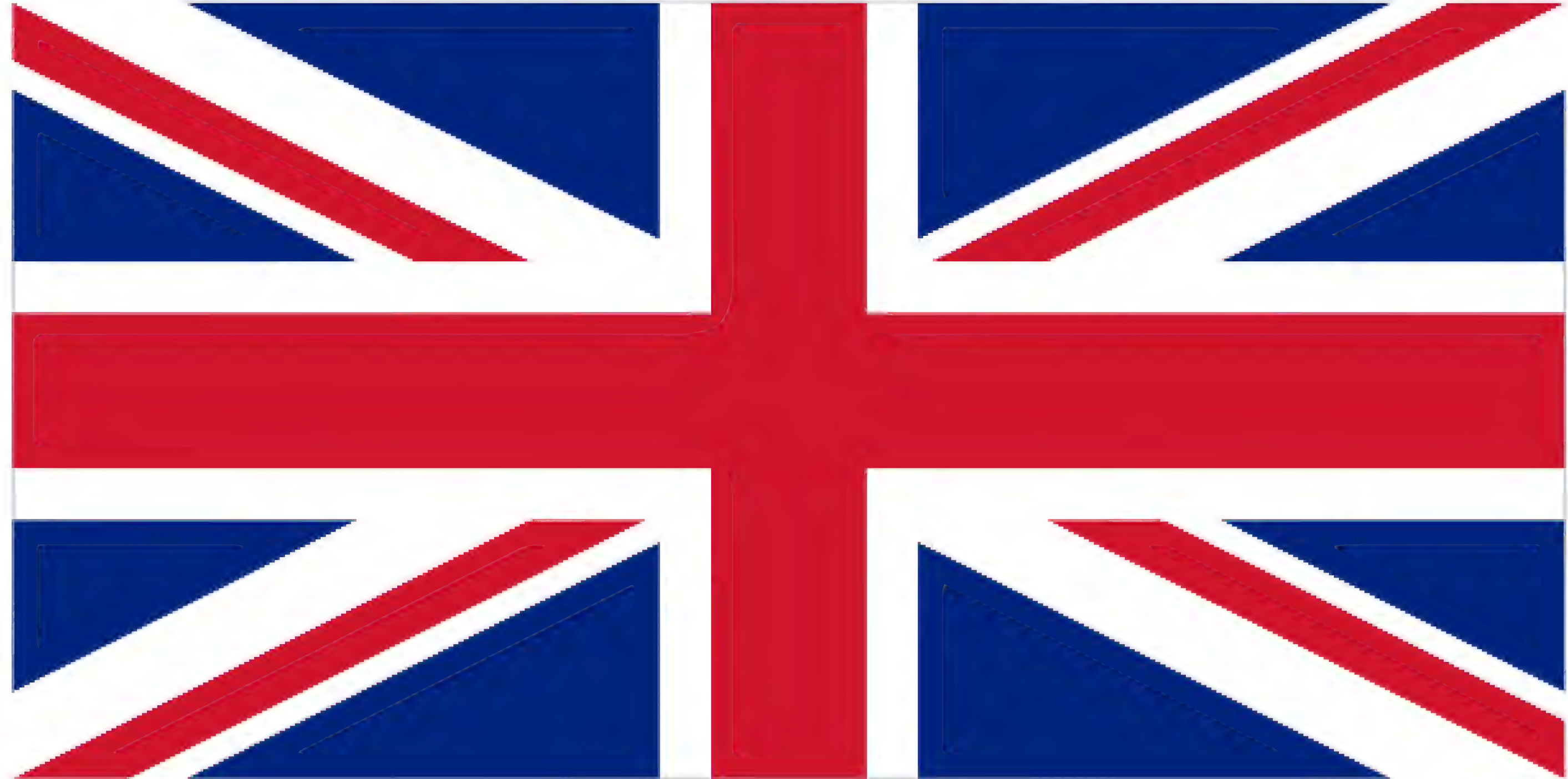
**Indians support English mathematicians!**



# ***Mathematical Foundations?***



© 2019 J. J. Crabtree [www.jonathancrabtree.com](http://www.jonathancrabtree.com)



# **TRUE!**

# **FALSE!**

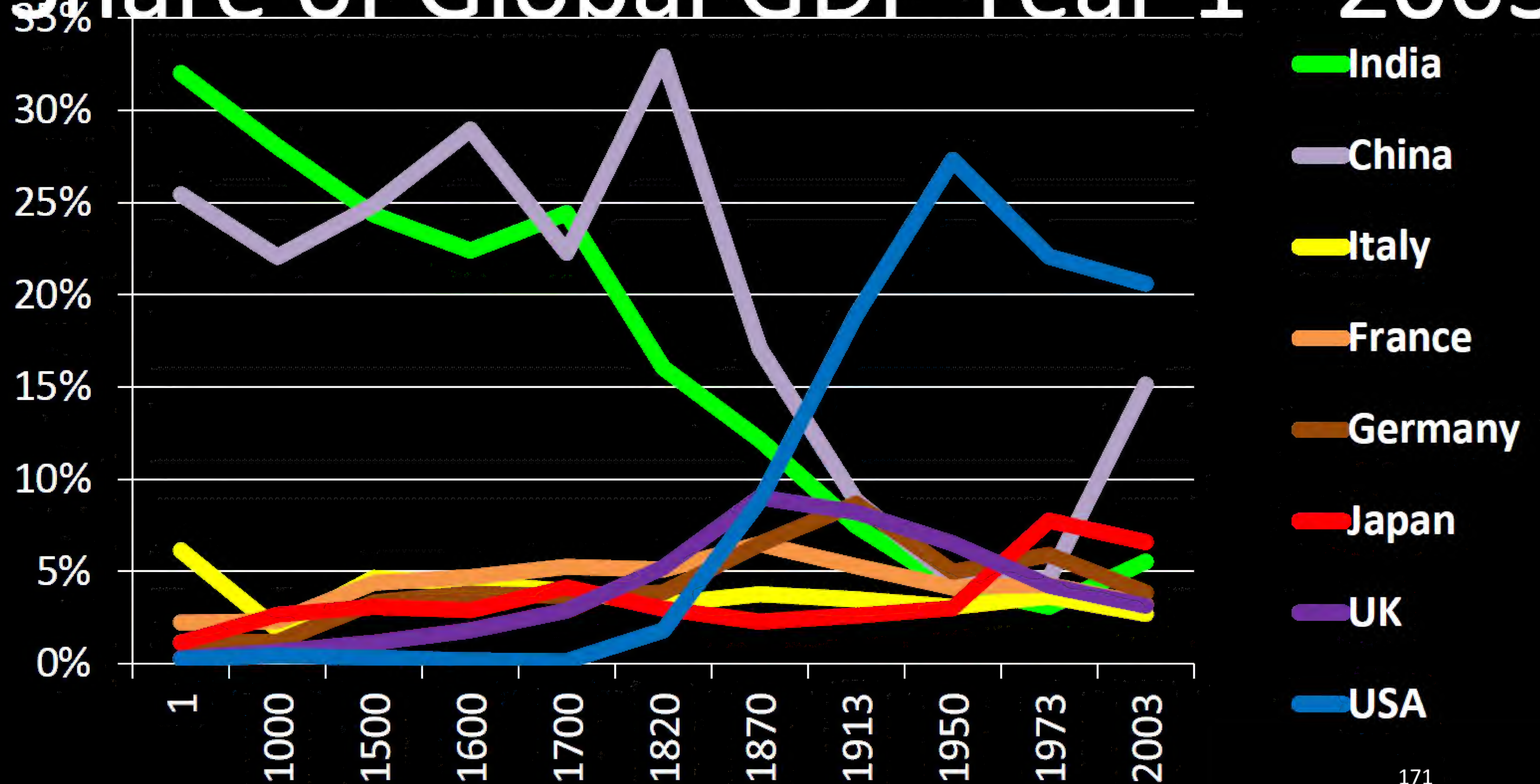




**India used to lead  
the world when it  
followed its own  
maths.**



# Share of Global GDP Year 1 - 2003





**The hard work has been done!**  
**India can update its**  
**maths and prosper**  
**or let this major maths**  
**education advantage pass by.**





**Thank you India for  
your gift of zero. It  
got lost, so I have  
returned it to you.**





**If you care,  
please share!**

**Thank you.**



# **PART 2. Negative & Positive Quantities on a Brahmaguptan Plane for India's Primary Classes**



**Jonathan J. Crabtree, Elementary Mathematics Historian, Australia.**

**9th National conference: Technology & Innovations in Math Education**  
**Biennial Conference of the Mathematics Department of the IIT Bombay.**

**Jointly organized by IISER, Pune & BATU, Lonere.**

**Pune India, 27<sup>th</sup> December 2019**



**“There are no studies demonstrating how negative numbers and algebra can be taught to such students in a meaningful way.”**





**“There are no studies demonstrating how negative numbers and algebra can be taught to such students in a meaningful way.”**



**Mathematics Education in India *Status and Outlook***

**Edited by R. Ramanujam and K. Subramaniam**

**Homi Bhabha Centre for Science Education**

**Tata Institute of Fundamental Research**



**“Negative numbers, usually introduced early in class 6, are known to be a problem area.”**



**Mathematics Education in India *Status and Outlook***

**Edited by R. Ramanujam and K. Subramaniam**

**Homi Bhabha Centre for Science Education**

**Tata Institute of Fundamental Research**



***“Negative times negative is positive is problematic to justify.”***



**A quote I jotted down from Dr. Shailesh Shirali  
this morning, 27 December.**



*“Negative times negative is positive is problematic to justify.”*



Now, Brahmagupta's simple idea of zero lets India's children understand  $-1 \times -1 = +1$  with absolute rock-solid certainty and intuitive conviction!



***“Something is amiss. Why are we adrift?  
What should be done?  
We need to gain insight from history.”***



**Some quotes I jotted down from Professor Dinesh Singh's inaugural TIME2019 address 26 December.**



***“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”***



**Some quotes I jotted down from Professor Dinesh Singh's inaugural TIME2019 address 26 December.**



*“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”*



**My goal is to answer questions like these and to give India's leaders and teachers the solutions children need.**



*“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”*



**To solve India’s primary-level mathematics education problems, I am creating a free ‘Sanskrit-based’ alternative to Arithmetic called Podometric™.**



***“Please forget everything you learned in school, because you have not learned it.”***



**Edmund Landau, *Foundations of Analysis***

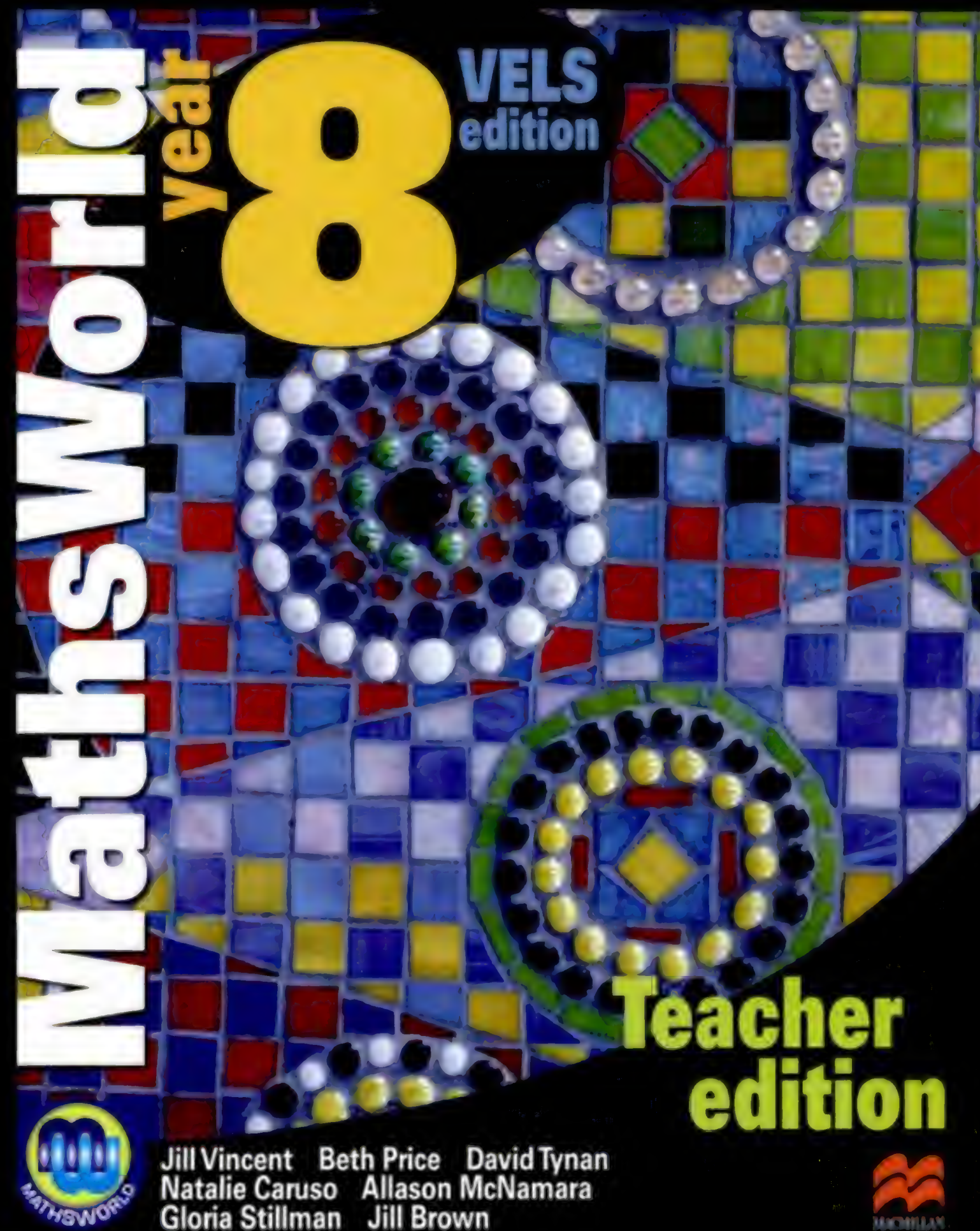


Please **forget** much about what you learned  
in school about **arithmetic**, because you  
**don't understand** my definition of **zero**.



*What Brahmagupta might say if he were alive today.*



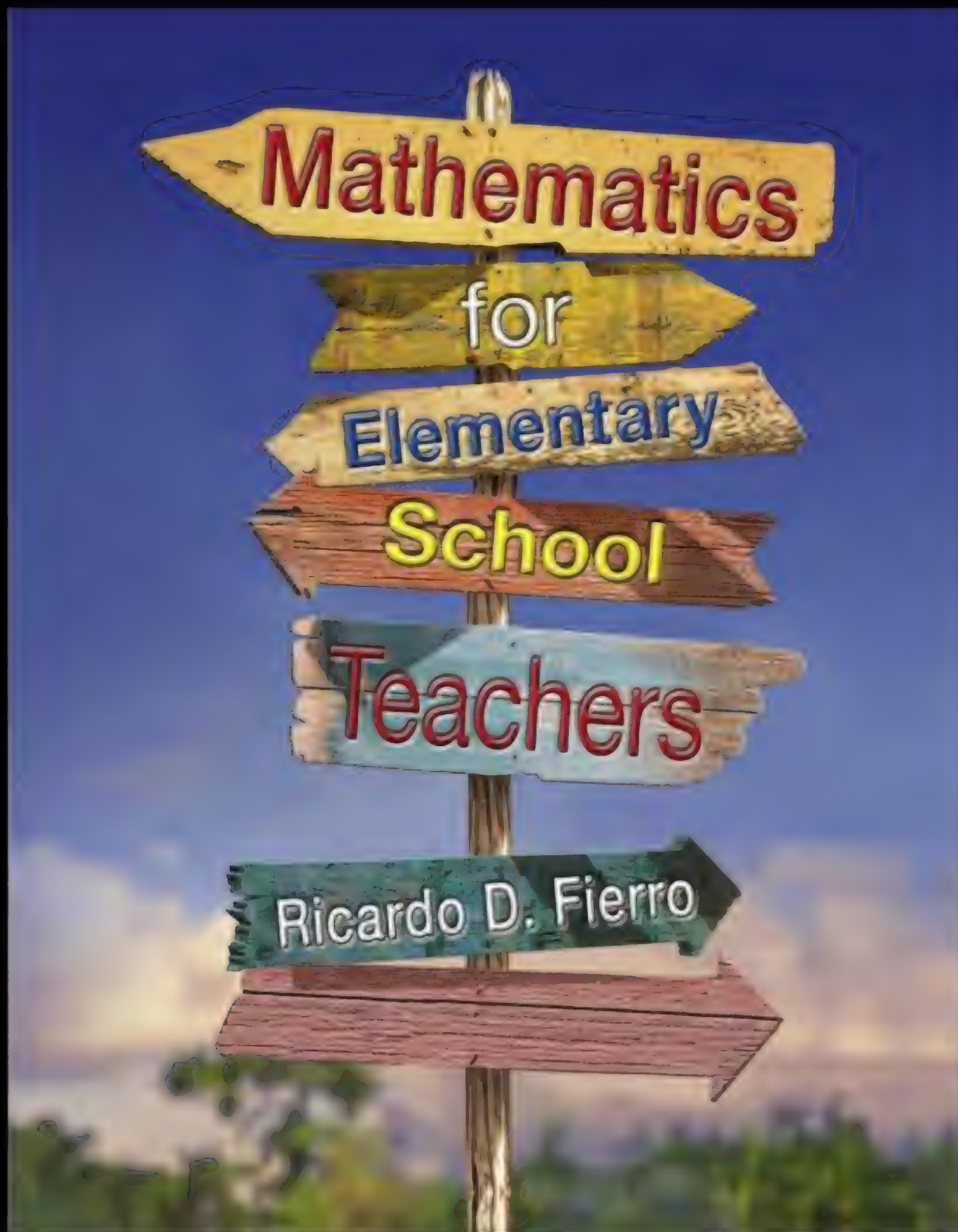


**“Brahmagupta... defined zero as the result of subtraction of a number from itself.”**

**“He also gave the following rules for operations on what he called ‘fortunes’ (positive numbers) and ‘debts’ (negative numbers).”**

**“The product... of two debts is one fortune.”**





**“Brahmagupta (598 – 670 CE) was a Hindu mathematician and astronomer who lived in the first century.”**

**“He used negative integers to represent debts and positive integers to represent assets.”**

**“The product ... of two debts is one fortune.”**



## Brahmagupta

"The product of a positive and a negative (number) is negative; of two negatives is positive; positive multiplied by 'positive is positive."

## Mahāvīra

"In the multiplication of two negative or two positive numbers the result is positive; but it is negative in the case of (the multiplication of) a positive and a negative number."

## Śrīpati

"On multiplying two negative or two positive numbers (the product is) positive; in the multiplication of positive and negative (the result is) negative."

## Bhāskara II

"The product of two positive or two negative (numbers) is positive; the product of positive and negative is negative." The same rule is stated by Nārāyaṇa.

**SOURCE:** *History of Hindu Mathematics: A Source Book, Part II, Algebra*, Bibhutibhusan Datta and Bidyāranya Avadesh Narayan Singh, pp. 22-23, Motilal Banarsidass, Lahore, 1938.



In 263 CE, Liu Hui wrote a commentary on the ancient  
***The Nine Chapters on the Mathematical Art***  
(九章算术 Jiǔzhāng Suànshù circa 100 CE).



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*I read the Nine Chapters as a boy, and studied it in full detail when I was older. I observed the division between the **dual natures of Yin and Yang** [the negative and positive aspects] which **sum up the fundamentals of mathematics**.*



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*The Nine Chapters on the Mathematical Art: Companion and Commentary,*  
Shen Kangshen, John N. Crossley and Anthony W. C. Lun, Oxford University Press, 2000.



If Maths is Language,  
it must obey the  
'Parts of Speech'  
of language.



# The Grammar of Arithmetic



Negative Seven

Minus

Negative Four?



Most adults answer  
*“Negative Eleven?”*



Most adults answer  
*“Negative Eleven?”*

**Why?**



Adjective Adjective  
Negative Seven

Verb

Minus

Adjective Adjective  
Negative Four?



Adjective Adjective  
Negative Seven  
Verb

Minus

Adjective Adjective  
Negative Four?

No Nouns!



Adjective Adjective  
Negative Seven  
Verb

Minus

Adjective Adjective  
Negative Four?

Adults struggle for rules memorized without  
meaning *'is it two negatives make a plus?'*



Seven Negatives

Minus

Four Negatives?



Every child answers  
“Three Negatives!”



Every child answers

**“Three Negatives!”**

**Why?**



Adjective Nouns  
Seven Negatives  
Verb

Minus

Adjective Nouns  
Four Negatives?



Adjective Nouns

Seven Negatives  
Verb

Minus

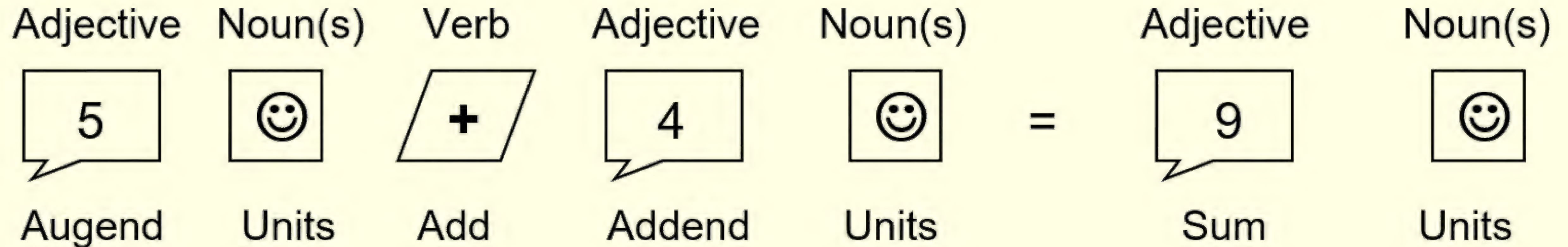
Adjective Nouns

Four Negatives?

Nouns Make Maths Meaningful!



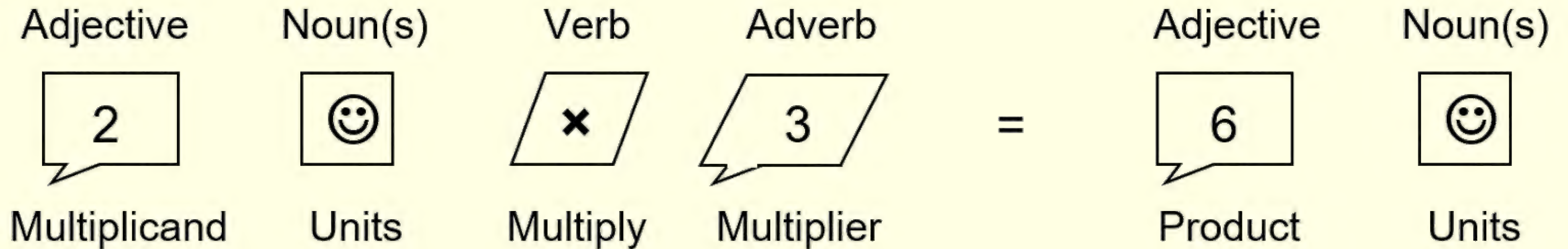
# Addition



[www.j.mp/IndiasMaths](http://www.j.mp/IndiasMaths)



# Multiplication



[www.j.mp/IndiasMaths](http://www.j.mp/IndiasMaths)



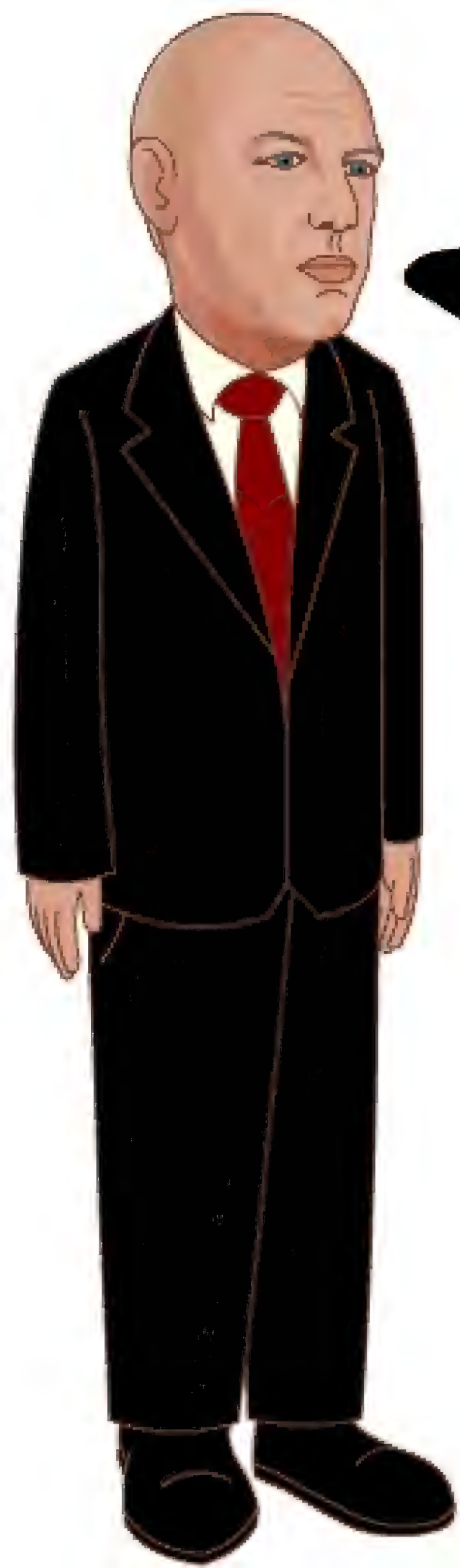
# Why the product of two negatives is positive

*A demonstration goes like this...*

Crabtree, Jonathan J. Snippet: A new reason negative multiplied by negative is positive Vinculum, Vol. 52, No. 3, Jul 2015

$1 + (-1) = 0$	Definition of $-1$ .
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$0 + (-1) \times (-1) = 1 + 0$	Definition of $-1$
$(-1) \times (-1) = 1$	Additive identity





**Brahmagupta defined zero**  
as the sum of a positive number  
and negative number of equal  
magnitude, सम-ऐक्यम् खम् (Brāhma  
Sphuta-siddhānta, Chapter 18:30a).

So **zero** was defined in India as:

$$+n + -n$$



2 धनयोर्धनमृणमृणयो-

3 र्धनार्णयोरन्तरं समैक्यं खम् ।

4 ऋणमैक्यं च धनमृणध-

5 नशून्ययोः शून्ययोः शून्यम् ॥ ३० ॥ (३१)

6 धनयोरैक्यं धनमृणयोरैक्यमृणं भवति । धनार्णयोरन्तरमेवैक्यं भव-  
7 ति । समयोर्धनार्णयोरैक्यं खं शून्यं भवति । ऋणशून्ययोरैक्यमृणं धनशू-  
8 न्ययोरैक्यं धनं शून्ययोरैक्यं च शून्यं भवति ।

9 अत्रोपपत्त्यर्थं मन्मुद्रिता भास्करबीजटिप्पणी द्रष्टव्या ॥ ३० ॥

10 इदानीं व्यवकलनमाह ।

11 ऊनमधिकाद्विशोध्यं धनं धनादृणमृणादधिकमूनात् ।

12 व्यस्तं तदन्तरं स्यादृणं धनं धनमृणं भवति ॥ ३१ ॥ (३२)

13 शून्यविहीनमृणमृणं धनं धनं भवति शून्यमाकाशम् ।

14 शोध्यं यदा धनमृणादृणं धनाद्वा तदा क्षेप्यम् ॥ ३२ ॥ (३३)

15 अधिकादुनादूनं धनं विशोध्यं शेषं धनं भवति । अधिकादृणादू-

16 नमृणं विशोध्यं शेषमृणं भवति । ऊनादुनादधिकं धनं वानादृणादधिक-

17 मृणं विशोध्यं तदा तदन्तरं व्यस्तं विपरीतं स्यात् । अर्थादधिकं धनं वि-

18 शोध्यं तदा शेषमृणं भवति । अधिकमृणं विशोध्यं तदा शेषं धनं भव-

19 ति । कथं विपरीतं भवतीत्याह । ऋणं धनं भवति धनं चणं भवतीति ।

20 चेदृणं शून्यविहीनं शून्येन विहीनं तदा ऋणं धनं च शून्यविहीनं धनं शून्यं

21 च शून्यविहीनमाकाशं शून्यं भवति । यदि ऋणादुनं शोध्यं वा धनादृणं

22 शोध्यं तदा क्षेप्यमर्थात् तदा तयोर्योग एवान्तरं भवतीति ।

23 अत्रोपपत्त्यर्थं मन्मुद्रिता भास्करबीजटिप्पणी विलोक्या ॥ ३१-३२ ॥

24 इदानीं गुणने करणसूत्रम् ।

25 ऋणमृणधनयोर्धातो धनमृणयोर्धनवधो धनं भवति ।

26 शून्यार्णयोः खधनयोः खशून्ययोर्धा वधः शून्यम् ॥ ३३ ॥ (३४)

27 ऋणधनयोर्धात ऋणं भवति । ऋणयोर्वधो धनवधो धनयोर्वधश्च

28 धनं भवति । शून्यार्णयोः खधनयोः शून्यधनयोर्धा खशून्ययोश्च वधः शून्यं

29 भवति ॥ ३३ ॥

30 इदानीं भागहारे करणसूत्रं वृत्तद्वयम् ।

31 धनभक्तं धनमृणहृतमृणं धनं भवति खं खभक्तं खम् ।

32 भक्तमृणेन धनमृणं धनेन हृतमृणमृणं भवति ॥ ३४ ॥ (३५)

33 खोद्धृतमृणं धनं वा तच्छेदं खमृणधनविभक्तं वा ।

34 ऋणधनयोर्वर्गः स्वं खं खस्य पदं कृतिर्यत् तत् ॥ ३५ ॥ (३६)

35 धनं धनभक्तं वा ऋणं ऋणभक्तं फलं धनं भवति । खभक्तं खं

36 फलं खं भवति । ऋणेन धनं भक्तं फलमृणं स्यात् । धनेन ऋणं हृतं फल-

37 मृणं भवति । ऋणं वा धनं खेनोद्धृतं तच्छेदं तस्य शून्यस्य छेदो यस्मि-

38 ऋणे वा धने तच्छेदं भवति । एवं खं शून्यमृणधनविभक्तं (शून्यं) वा त-

39 छेदं भवति । फलं शून्यं भवति वा शून्यं तद्वरं स्यादित्यर्थः । ऋणधन-

40 योर्वर्गः स्वं भवति । खस्य वर्गः खं भवति । तदेव वर्गस्य पदं भवति

41 यत्कृतिः स एव वर्गो भवेदिति । भास्करबीजेऽप्येतदेव सर्वम् । अत्र

42 खभक्तं खमर्थात् ० इदं सर्वदा शून्यसमं नेत्येतदर्थं चलनकलनं विलो-

43 क्यम् ॥ ३४-३५ ॥

44 इदानीं सङ्क्रमणविषमकर्माह ।

45 योगोऽन्तरयुतहीनो द्विहृतः सङ्क्रमणमन्तरविभक्तं वा ।

46 वर्गान्तरमन्तरयुतहीनं द्विहृतं विषमकर्म ॥ ३६ ॥ (३७)

47 योगो राश्यायोगोऽन्तरेण राश्यन्तरेण युतो हीनश्च द्विहृतो दलि-

48 तो राशी स्तः । इदं सङ्क्रमणं नाम गणितम् । वा राश्यावर्गान्तरं राश्य-



# Brahmagupta's 5 Addition Laws

- AL1** positive plus positive is positive
- AL2** negative plus negative is negative
- AL3** positive plus negative is the difference between the positive and negative
- AL4** when positive and negative are equal the sum is zero
- positive plus zero is positive
- AL5** negative plus zero is negative
- zero plus zero is zero

# Brahmagupta's 4 Multiplication Laws

- ML1** The product of a negative and a positive is negative.
- ML2** The product of two negatives is positive.
- ML3** The product of two positives is positive.
- The product of zero and a negative,  
**ML4** of zero and a positive, or  
of two zeros is zero.

# Brahmagupta's 5 Subtraction Laws

- SL1** A smaller positive subtracted from a larger positive is positive.
- SL2** A smaller negative subtracted from a larger negative is negative.
- SL3** If a larger negative or positive is to be subtracted from a smaller negative or positive, the sign of their difference is reversed – negative becomes positive and positive negative.
- SL4** A negative minus zero is negative,  
a positive minus zero is positive,  
zero minus zero is zero.
- SL5** When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

# Brahmagupta's 4 Division Laws

- DL1** A positive divided by a positive is positive.
- DL2** A negative divided by a negative is positive.
- DL3** A positive divided by a negative is negative.
- DL4** A negative divided by a positive is negative.



# Seeing a Simple Proof for Brahmagupta's ML2

*The product of two 'dashed numbers' is positive.*

Via Brahmagupta we multiply **positives** and **negatives** by either adding to **zero** multiple times or subtracting from **zero** multiple times.





# Seeing a Simple Proof for Brahmagupta's ML2

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$-a \times +b =$   
 *$-a$  added to*  
*zero  $b$  times*



$-a \times -b =$   
 *$-a$  subtracted from*  
*zero  $b$  times*



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 *$-a$  subtracted from  
zero  $b$  times*

With integral multiplication...

$a$  is the **adjective** describing counts or measures of **noun** quantities  
 $b$  is the **adverb** describing **verb** counts of additions or subtractions



Seeing a Simple Proof for Brahmagupta's ML2  
*The product of two 'dashed numbers' is positive.*

$-a \times -b = -a$  subtracted from zero  $b$  times

$-1 \times -1 = -1$  subtracted from zero 1 time





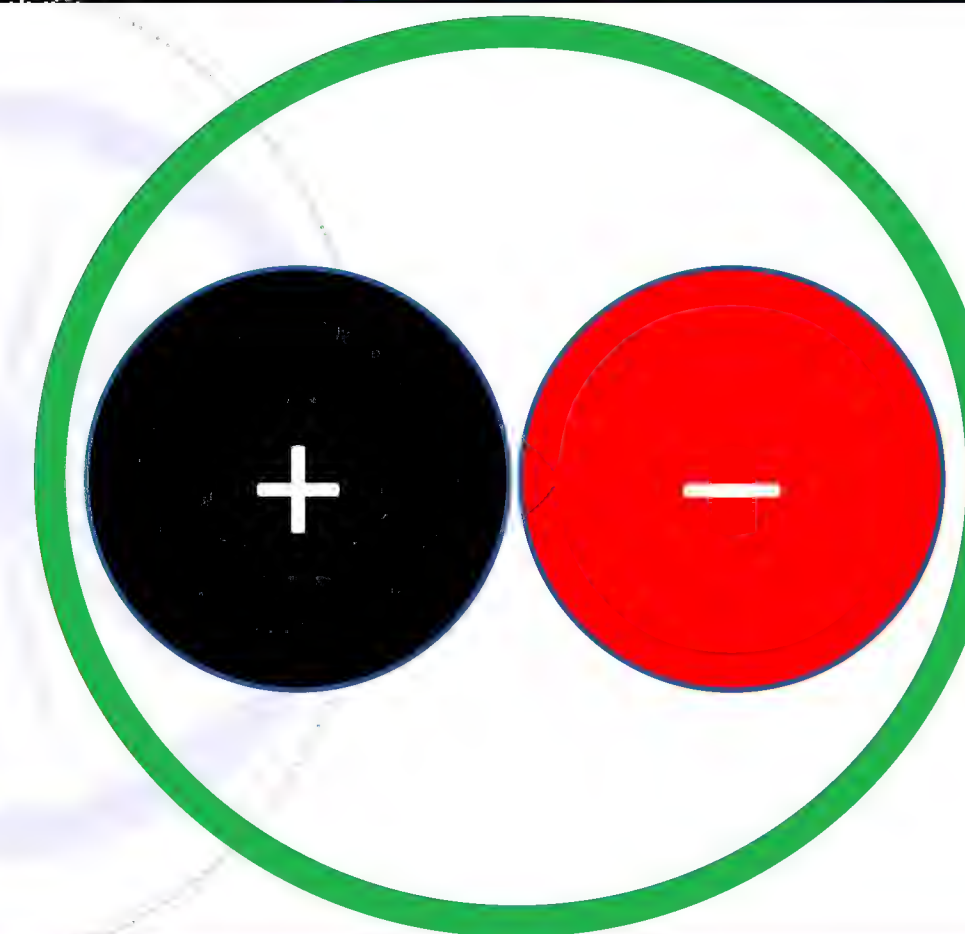
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$-1 \times -1 = -1$  subtracted from zero 1 time

$+1 + -1$



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$0 + (-1) \times (-1) = 1 + 0$	Definition of $-1$
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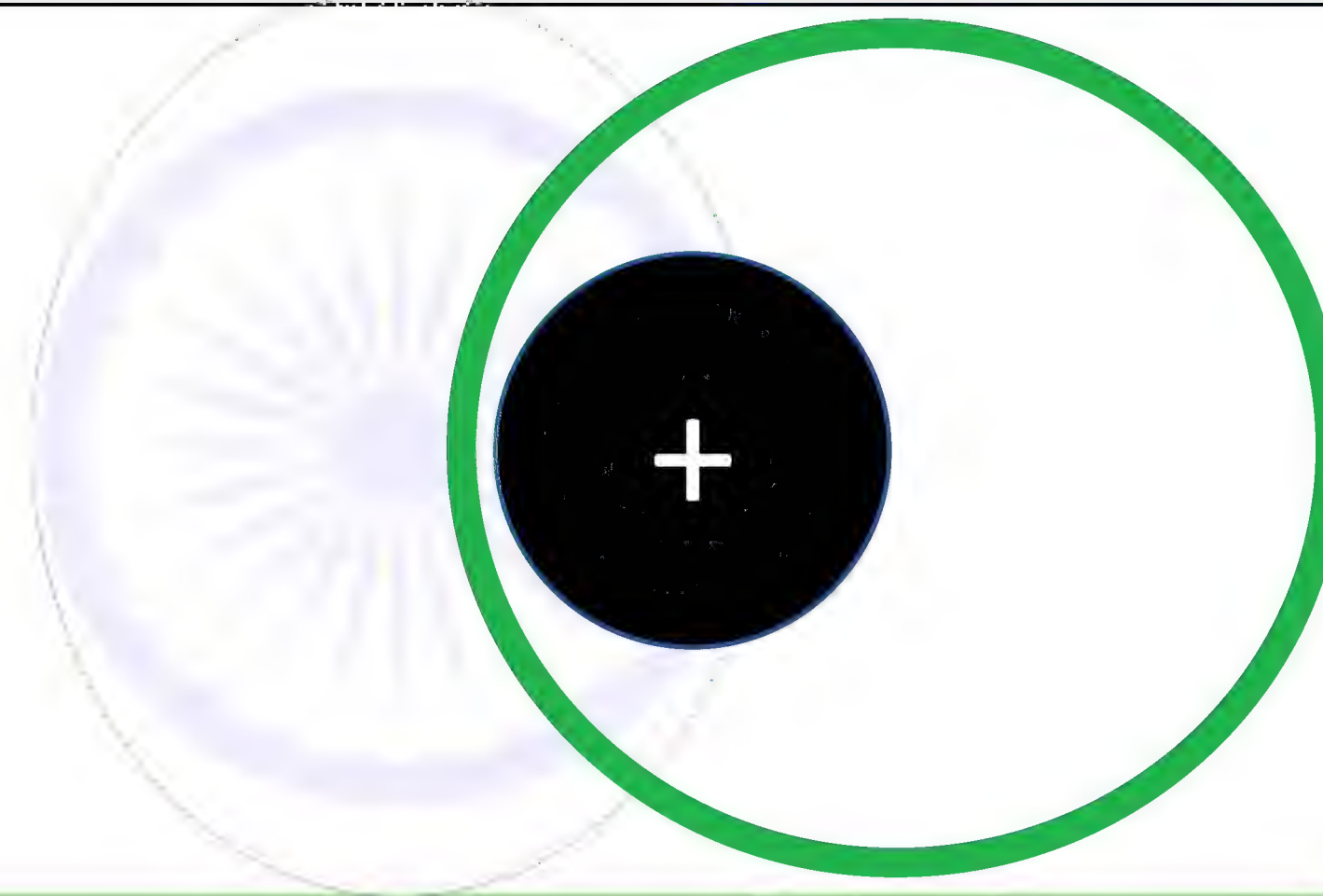
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$-1 \times -1 = -1$  subtracted from zero 1 time

$$-1 \times -1 = +1$$



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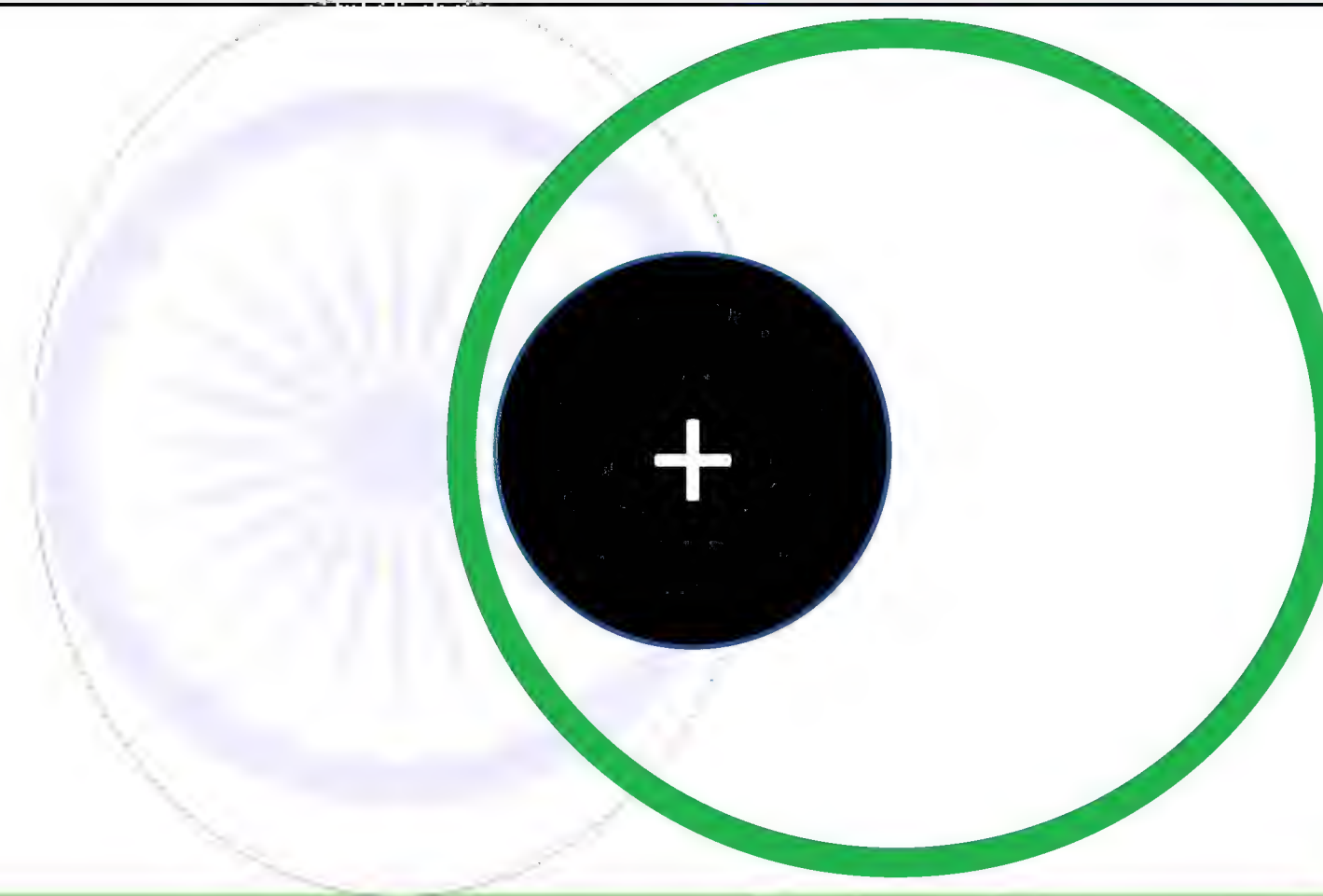
My Sanskrit-Based Demonstration  
i.e.  $-1$  subtracted 1 time from 0 =  $+1$



Seeing a Simple Proof for Brahmagupta's ML2  
*The product of two 'dashed numbers' is positive.*

$-a \times -b = -a$  subtracted from zero  $b$  times  
 $-1 \times -1 = -1$  subtracted from zero 1 time

$$-1 \times -1 = +1$$



$1 + (-1) = 0$	Definition of $-1$ .
$-1 \times [1 + (-1)] = -1 \times 0$	Both sides multiplied by $-1$ .
$(-1) \times 1 + (-1) \times (-1) = 0$	Distributive law
$(-1) + (-1) \times (-1) = 0$	Multiplicative identity
$1 + [(-1) + (-1) \times (-1)] = 1 + 0$	Add both sides to 1.
$[1 + (-1)] + (-1) \times (-1) = 1 + 0$	Associative law
$0 + (-1) \times (-1) = 1 + 0$	Definition of $-1$
$(-1) \times (-1) = 1$	Additive identity

My Sanskrit-Based Demonstration  
i.e.  $-1$  subtracted 1 time from 0 =  $+1$

My Western  
Demonstration



**With our corrected understanding of Integer arithmetic, we can now depict it on a Brahmaguptan Plane.**



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*n* times added to 0  
multiplier

Q. II

Q. I

$+n$   
↑

$-a \times +n = \text{Negative Product}$   
i.e. a debt added *n* times  
to zero.       $0 + -a + -a + \dots$

$+a \times +n = \text{Positive Product}$   
i.e. a fortune added *n* times  
to zero.       $0 + +a + +a + \dots$

$-a$  ←  
multiplicand  
e.g. debt

→  $+a$   
multiplicand  
e.g. fortune



$-a$  ← **multiplicand**  
 e.g. debt

**multiplicand**  
 e.g. fortune →  $+a$

$-a \times -n = \text{Positive Product}$   
 i.e. a debt subtracted  $n$  times  
 from zero.  $0 - -a - -a + \dots$

**Q. III**

$+a \times -n = \text{Negative Product}$   
 i.e. a fortune subtracted  $n$  times  
 from zero.  $0 - +a - +a + \dots$

**Q. IV**

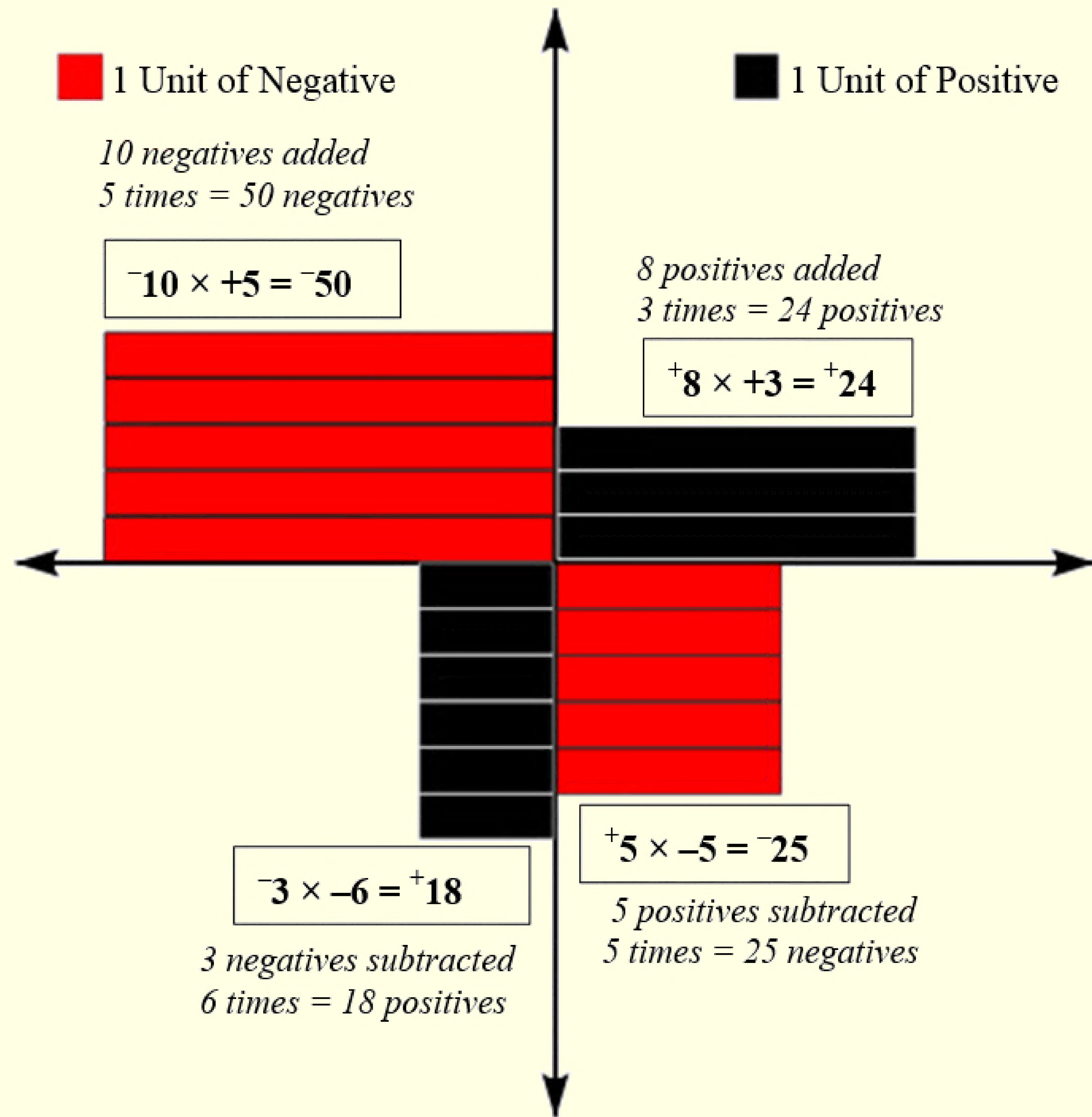
$-n$

**multiplier**

**$n$  times subtracted from 0**



# The Brahmaguptan Plane with both positive and negative areas.





Jonathan J Crabtree's 'Brahmaguptan Plane' with positive and negative areas.

Addition of Integers to Zero

Negatives Added N Times to Zero									Multiplier	Positives Added N Times to Zero								
81	72	63	54	45	36	27	18	9		9	18	27	36	45	54	63	72	81
72	64	56	48	40	32	24	16	8		8	16	24	32	40	48	56	64	72
63	56	49	42	35	28	21	14	7		7	14	21	28	35	42	49	56	63
54	48	42	36	30	24	18	12	6		6	12	18	24	30	36	42	48	54
45	40	36	30	24	18	12	6	5		5	10	15	20	25	30	35	40	45
36	32	28	24	18	12	9	6	4		4	8	12	16	20	24	28	32	36
27	24	21	18	15	12	9	6	3		3	6	9	12	15	18	21	24	27
18	16	14	12	10	8	6	4	2		2	4	6	8	10	12	14	16	18
9	8	7	6	5	4	3	2	1		1	2	3	4	5	6	7	8	9
-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9
9	8	7	6	5	4	3	2	1	-1	1	2	3	4	5	6	7	8	9
18	16	14	12	10	8	6	4	2	-2	2	4	6	8	10	12	14	16	18
27	24	21	18	15	12	9	6	3	-3	3	6	9	12	15	18	21	24	27
36	32	28	24	20	16	12	8	4	-4	4	8	12	16	20	24	28	32	36
45	40	36	30	24	18	12	6	5	-5	5	10	15	20	25	30	35	40	45
54	48	42	36	30	24	18	12	6	-6	6	12	18	24	30	36	42	48	54
63	56	49	42	35	28	21	14	7	-7	7	14	21	28	35	42	49	56	63
72	64	56	48	40	32	24	16	8	-8	8	16	24	32	40	48	56	64	72
81	72	63	54	45	36	27	18	9	-9	9	18	27	36	45	54	63	72	81
Negatives Subtracted N Times from Zero									Multiplier	Positives Subtracted N Times from Zero								

Subtraction of Integers from Zero



# Representations of Negative and Positive Quantities on a 'Brahmaguptan Plane' for India's Primary Classes

*Jonathan J Crabtree*

[www.j.mp/BrahmaguptanPlane](http://www.j.mp/BrahmaguptanPlane)

**Abstract:** *Children's fear of maths is often associated with the introduction of negative numbers. By way of example, asking adult non-mathematicians for the answer to 'negative seven minus negative four' usually results in a wrong answer. However, asking the same question to 12-year-old children in the form What does seven negatives minus four negatives equal? usually results in the right answer. Why is the difference in comprehension so dramatic? In the problematic expression negative seven minus negative four the syntactic structure is adjective adjective verb adjective adjective. With the absence of a noun, the meaning of such maths for most children is lost. Instead, children (and adults) cling to rules memorised without meaning, such as 'two minuses make a plus'. So, what can we do? The answer is simple. We return to 7<sup>th</sup> Century writings of India, where we discover the astronomer Brahmagupta documented 'adjective-noun' style laws of sign, not for abstract numbers, but for positive quantities, negative quantities and zero. With this insight, we depict simple object-oriented representations of integer arithmetic involving positive and negative quantities. Such a quantitative pedagogy is concrete in nature, yet isomorphic to 'signed numbers.' Therefore, a solid intuitive foundation of integer arithmetic can be laid. Upon this foundation more abstract structures can be built. The integer teaching model that emerges is called the 'Brahmaguptan Plane'.*



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